

GRACE/GRACE-FO: Gravity Recovery and Climate Experiment

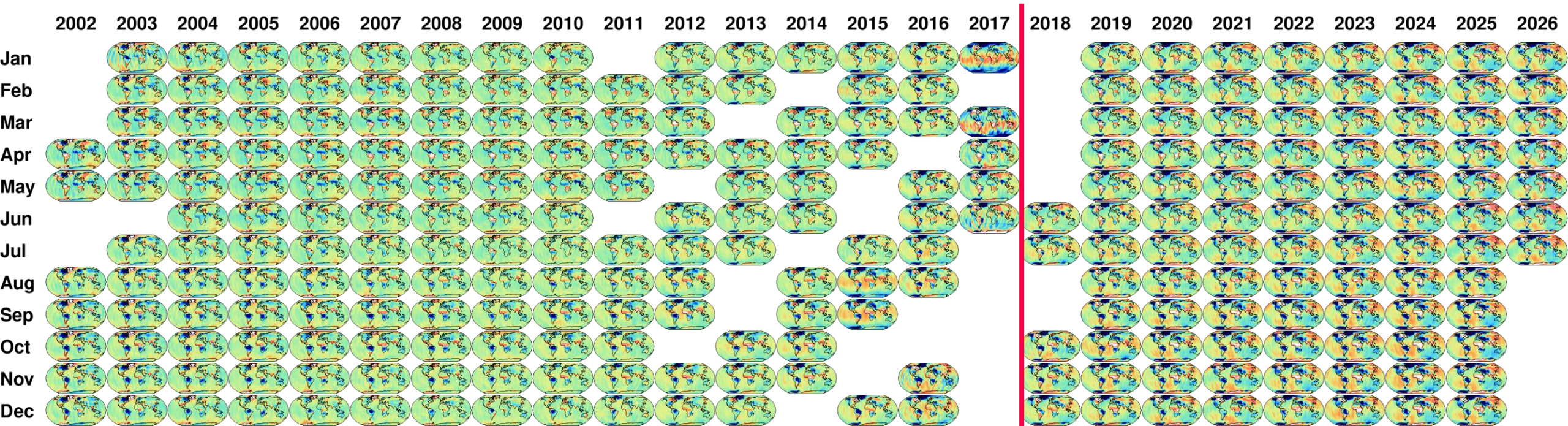
Torsten Mayer-Gürr

Institute of Geodesy
Graz University of Technology

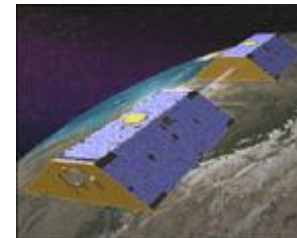
Parameterschätzung Übung
2026-10-21



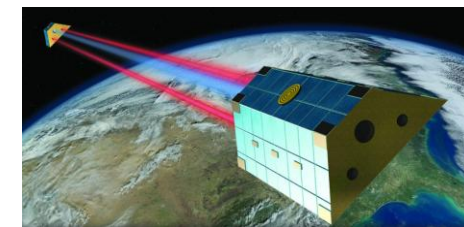
ITSG-Grace monthly gravity field solutions



GRACE

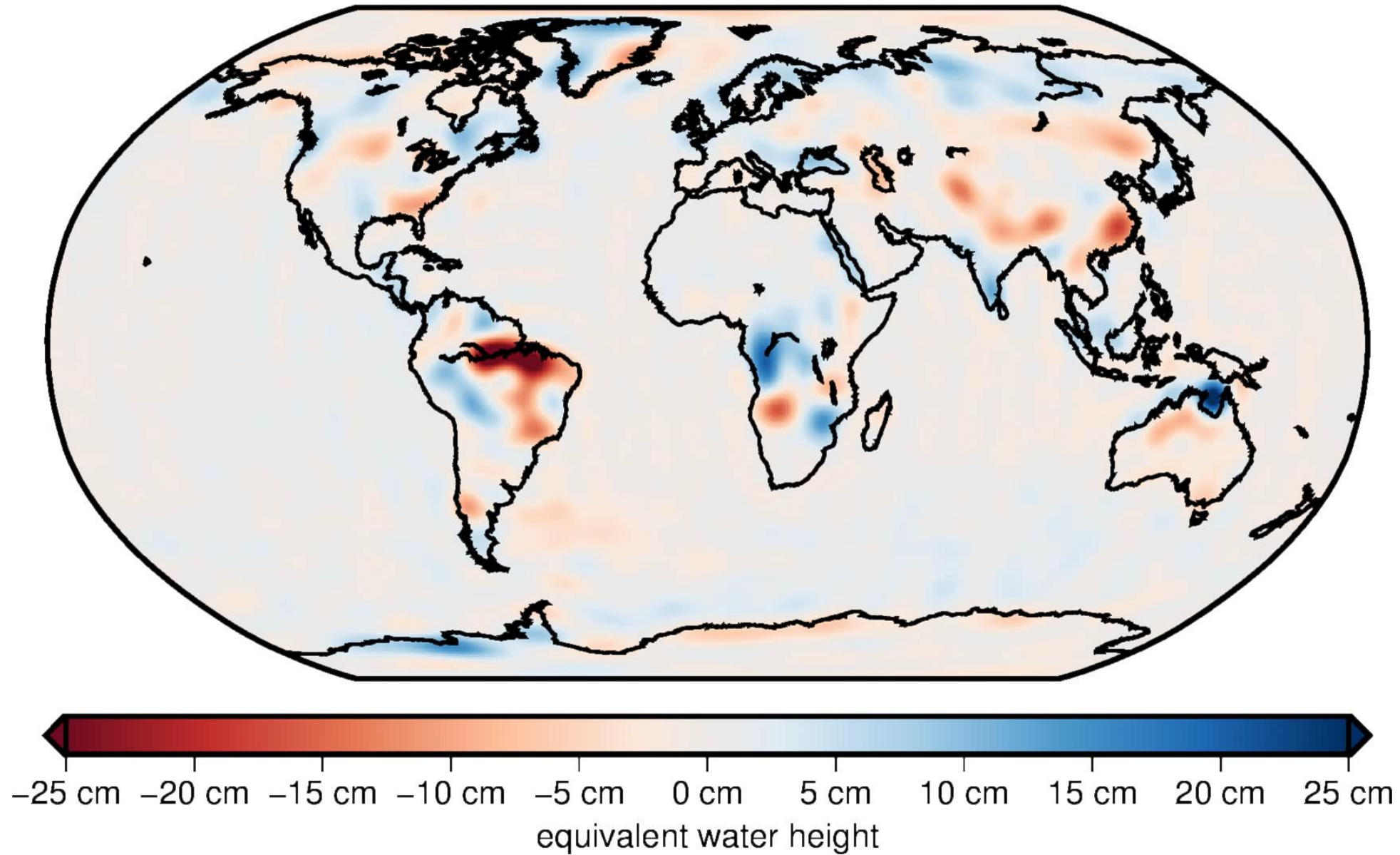


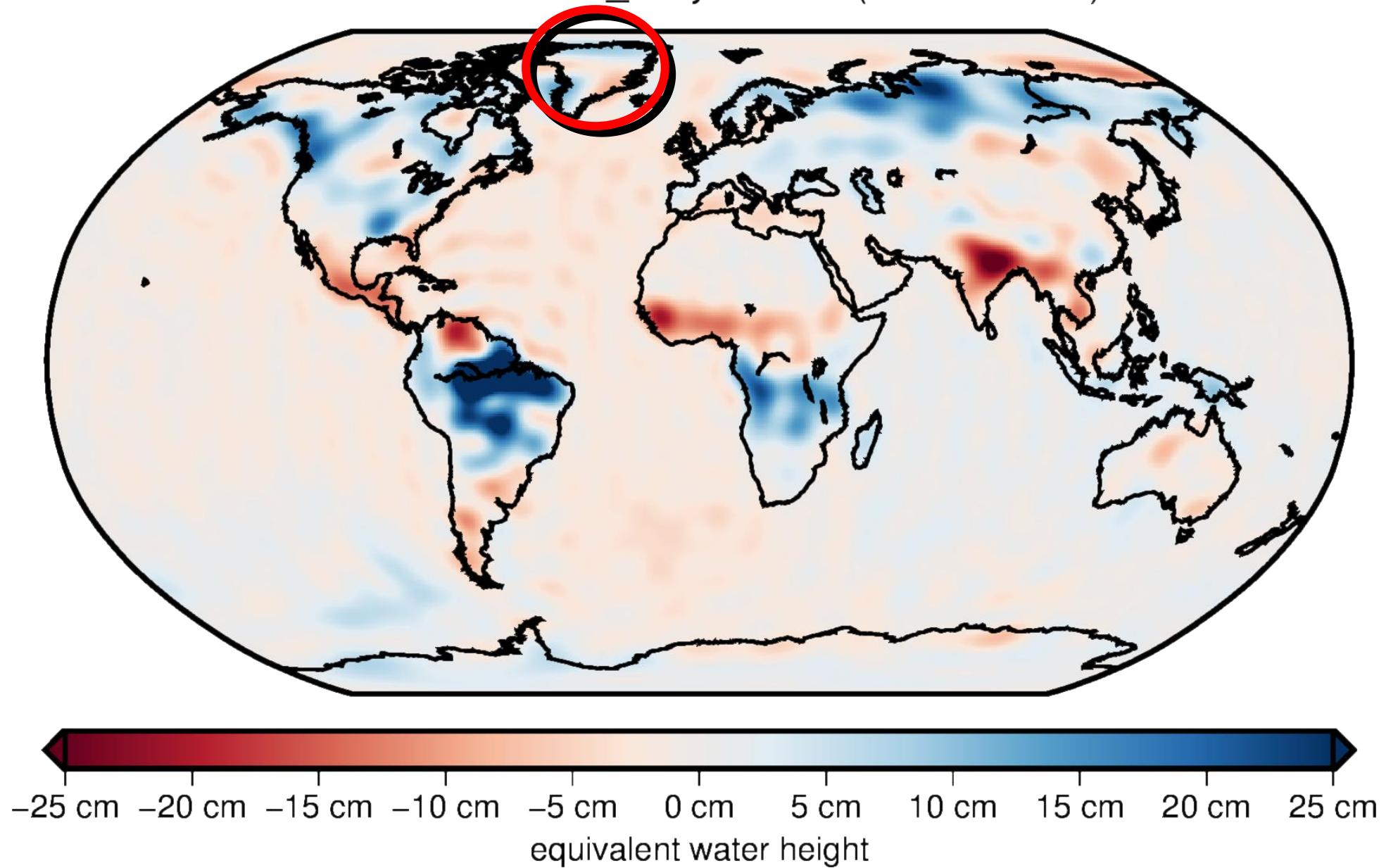
GRACE-FO



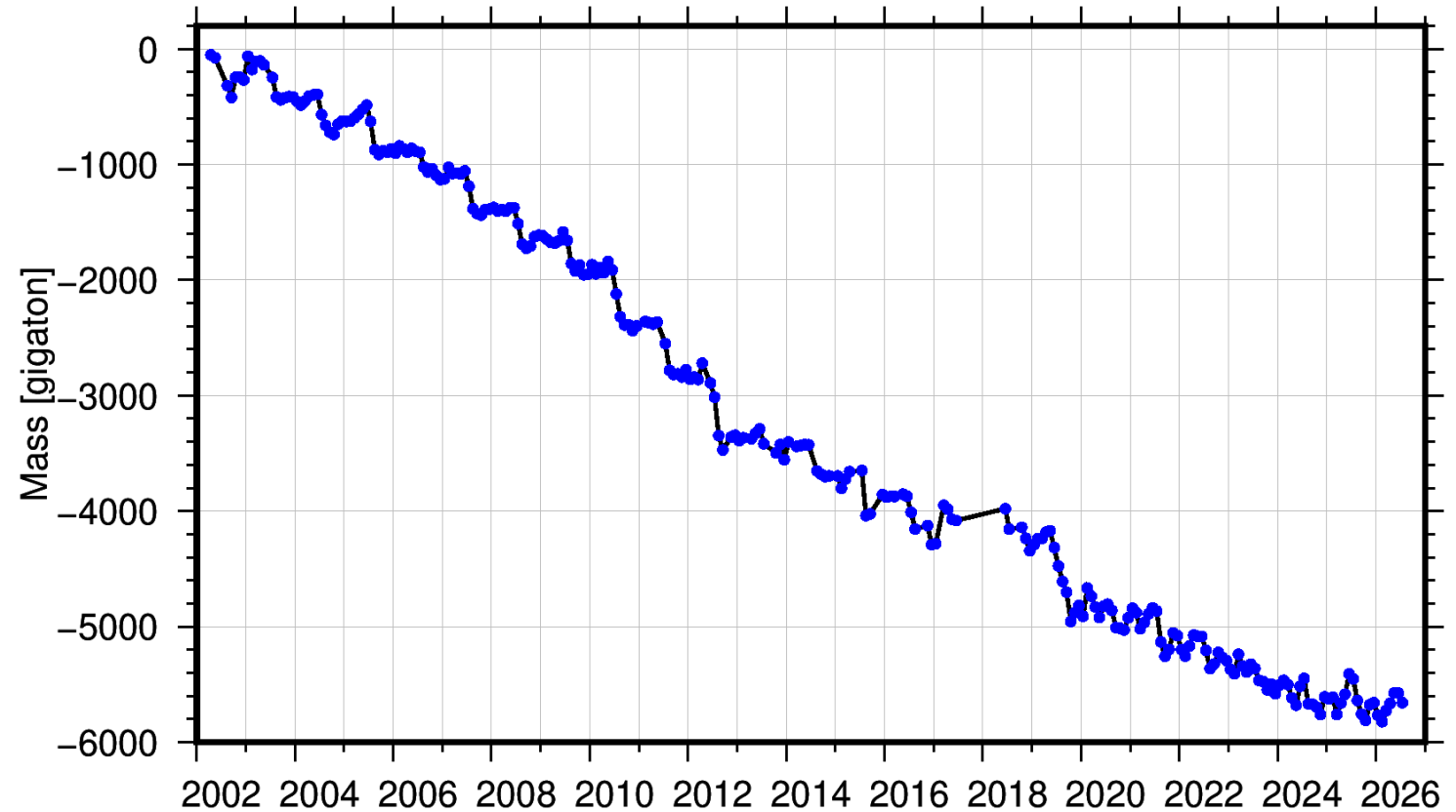
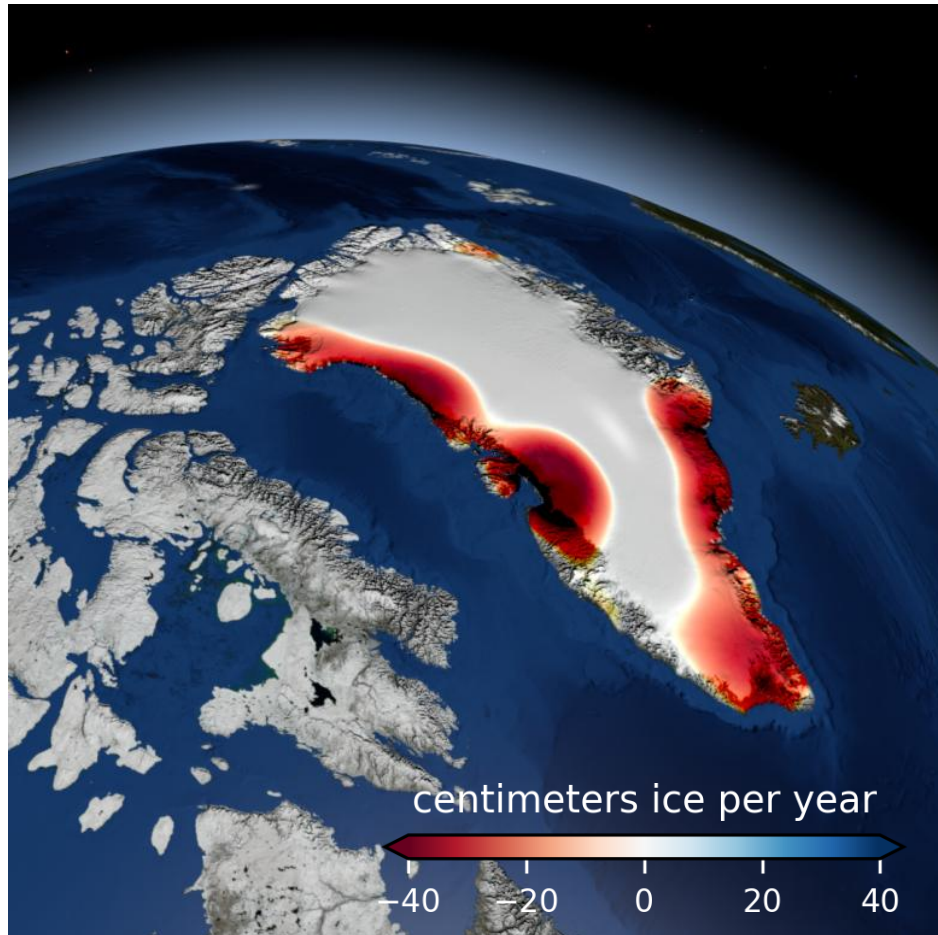
ITSG-Grace2018: Monthly and Daily solutions

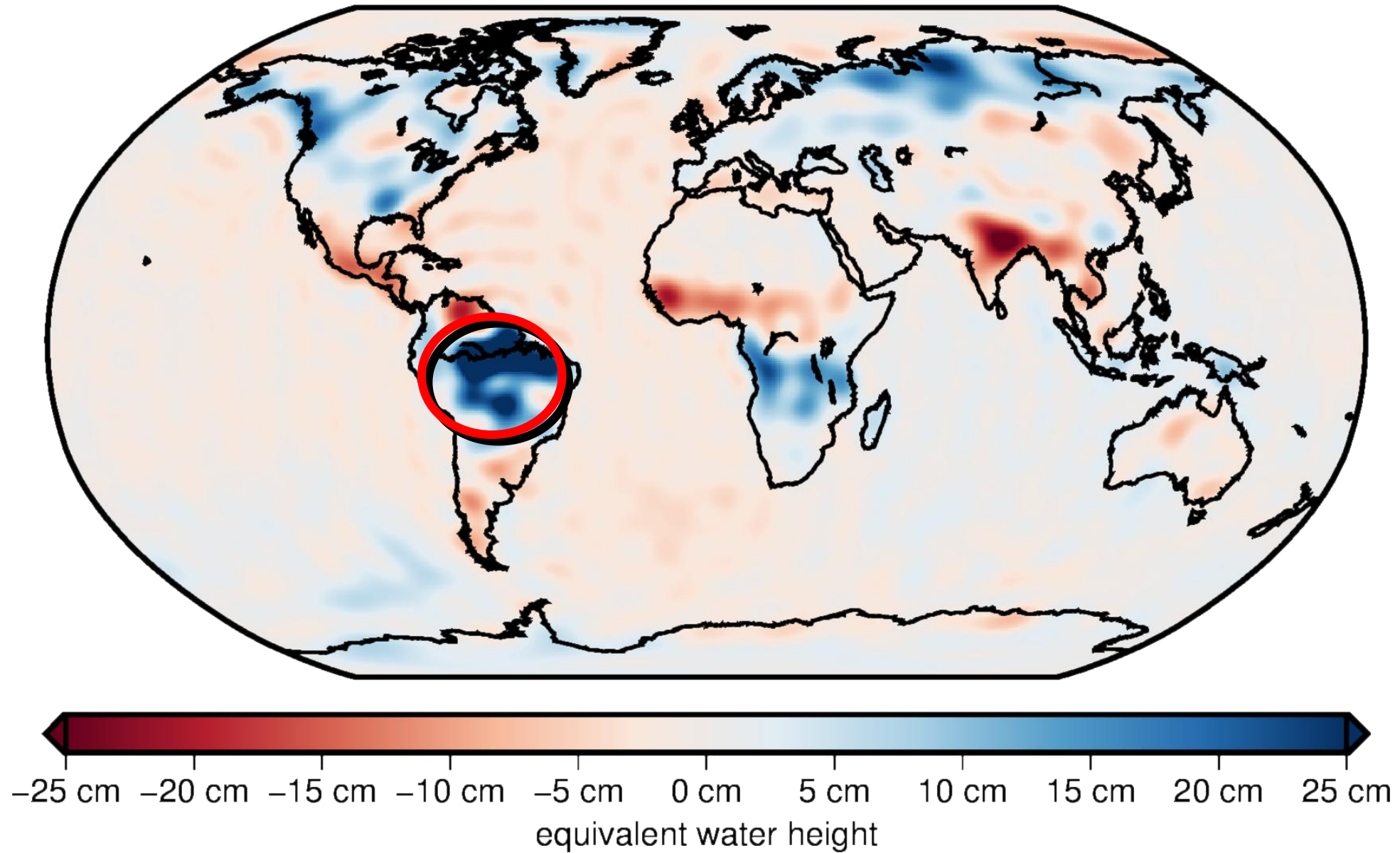
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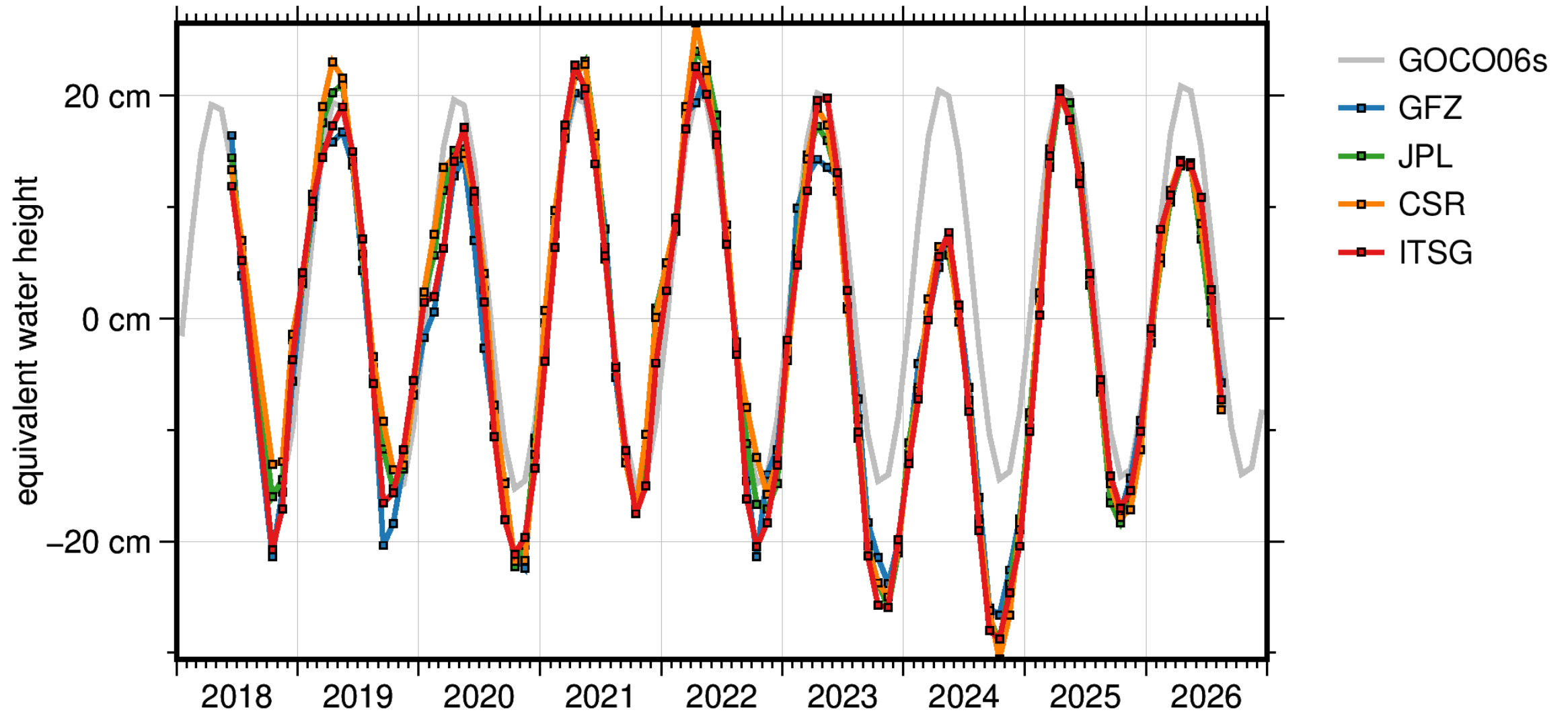


Ice melting in Greenland

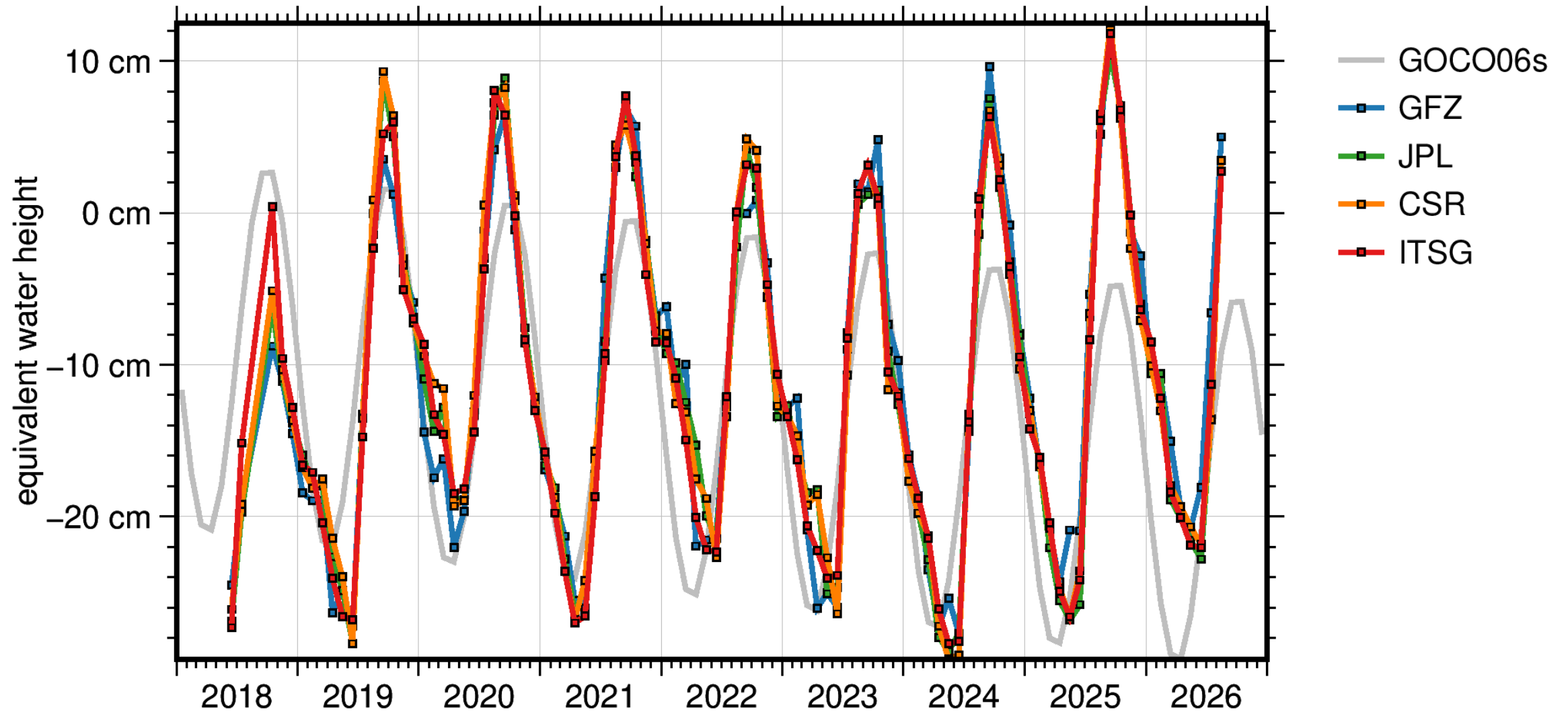




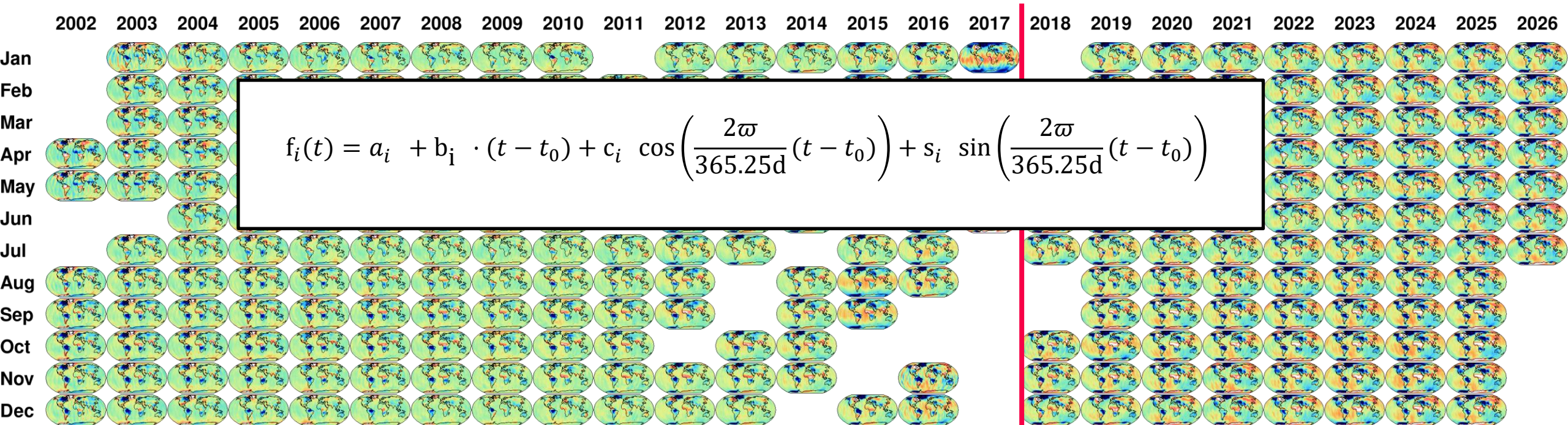
amazon, Gauss 300km



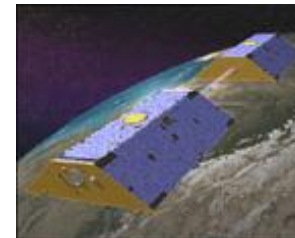
ganges, Gauss 300km



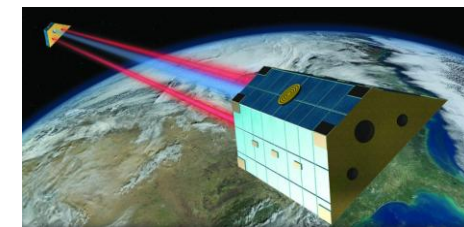
ITSG-Grace monthly gravity field solutions



GRACE

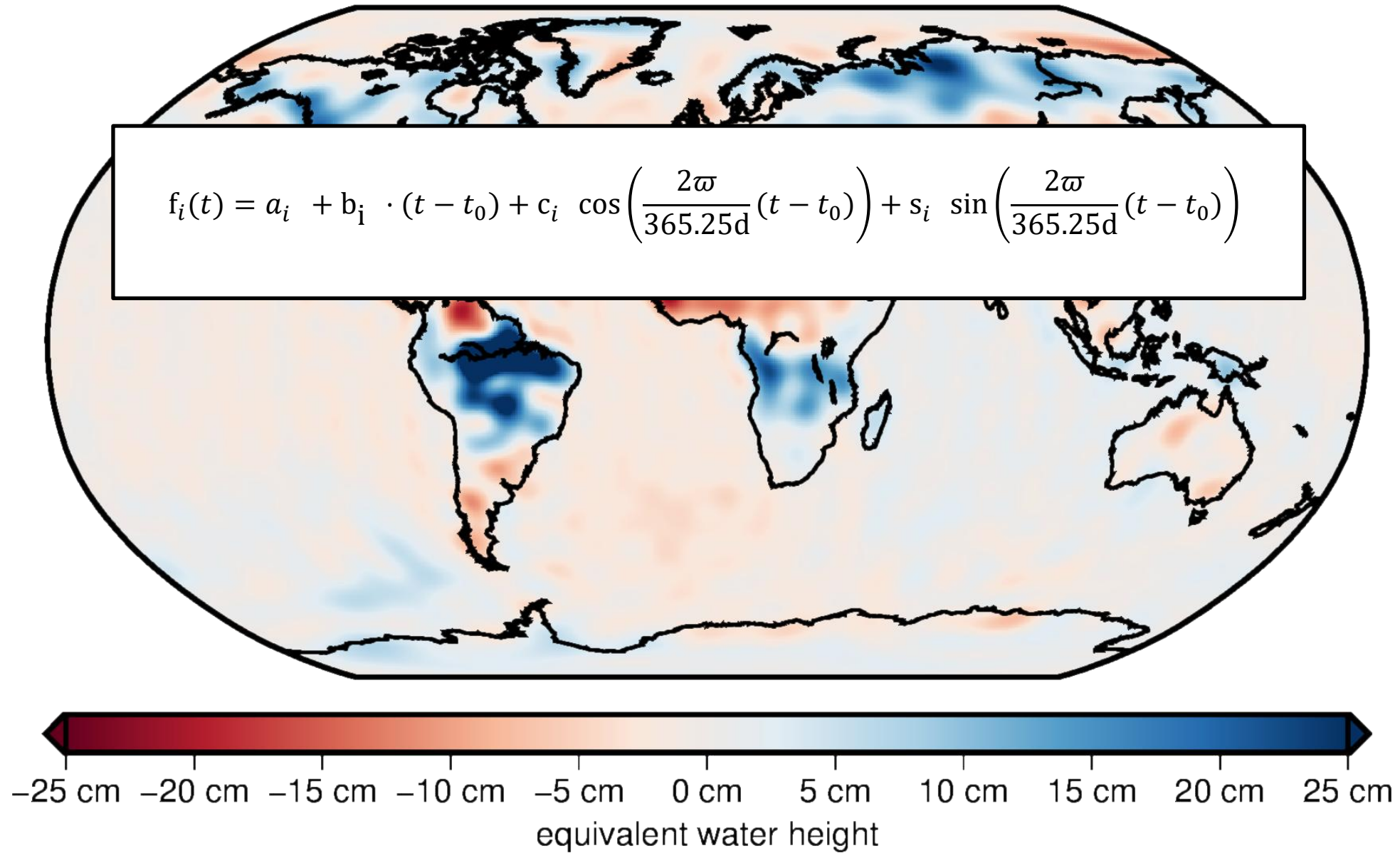


GRACE-FO



ITSG-Grace2018: Monthly and Daily solutions

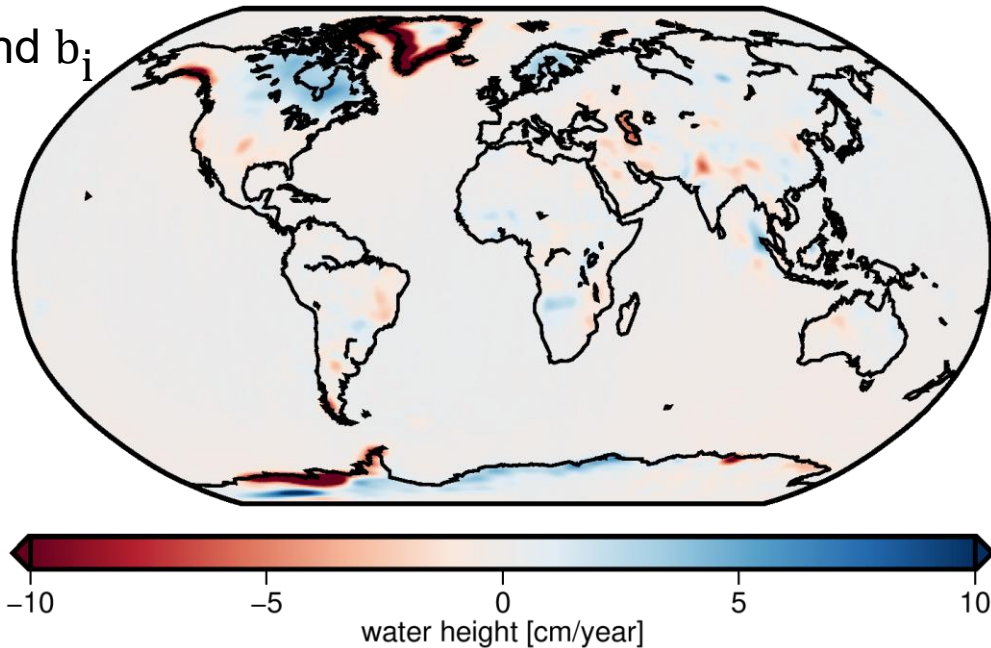
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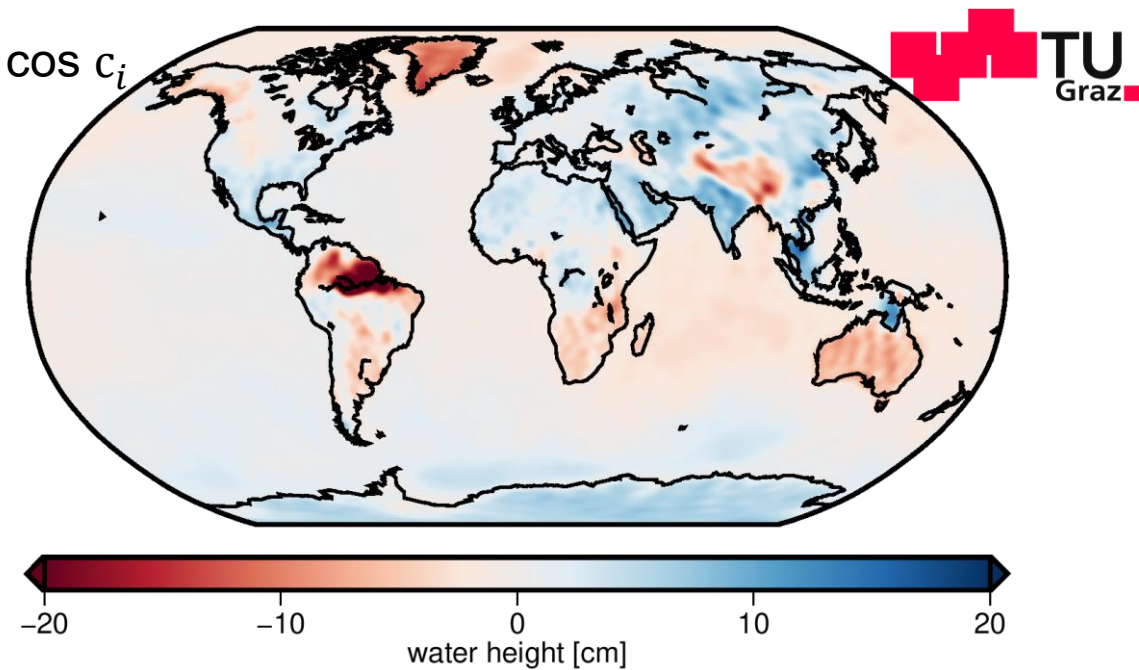
Trend and annual estimation

$$\begin{aligned} f_i(t) = & a_i \\ & + b_i \cdot (t - t_0) \\ & + c_i \cos\left(\frac{2\pi}{365.25d}(t - t_0)\right) \\ & + s_i \sin\left(\frac{2\pi}{365.25d}(t - t_0)\right) \end{aligned}$$

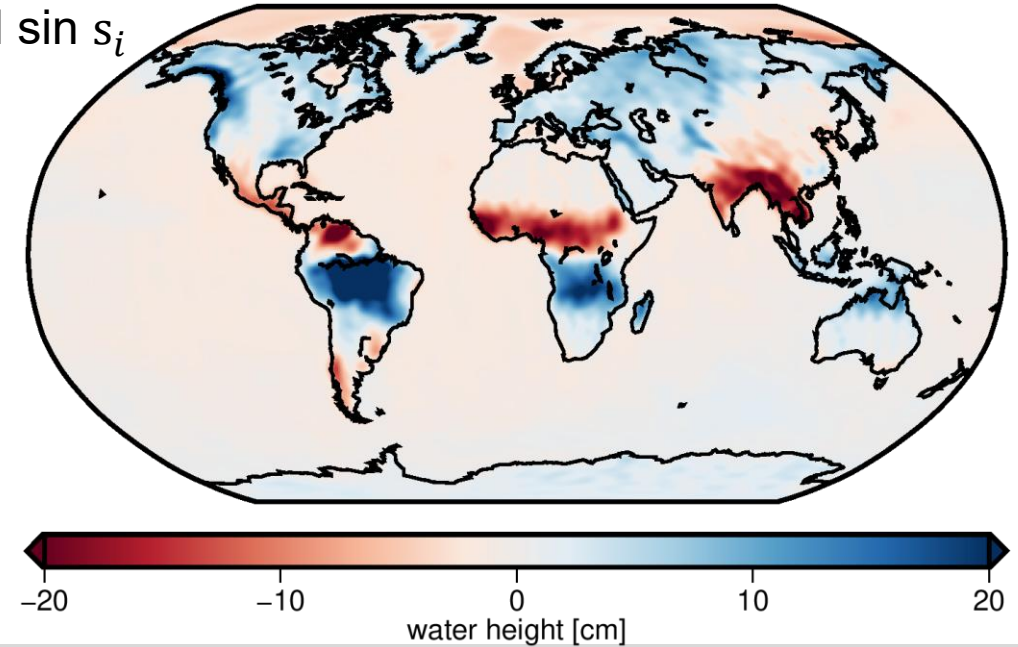
Trend b_i



Annual $\cos c_i$



Annual $\sin s_i$



Jährliches Signal

- Jährliche Schwingung

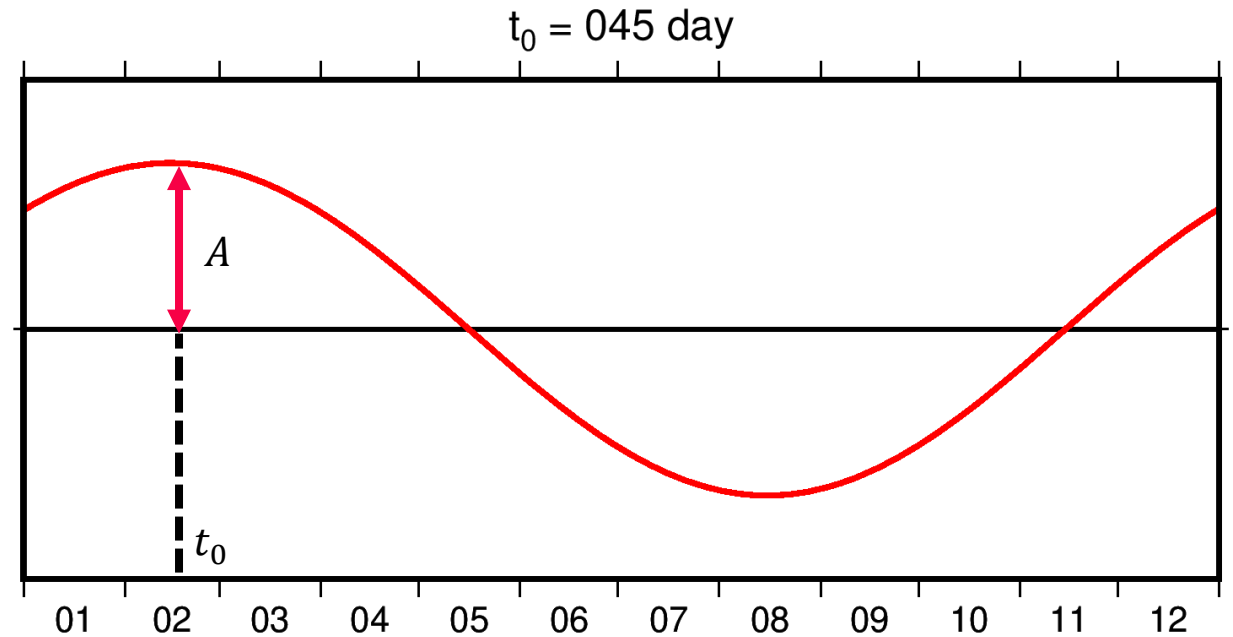
$$f(t) = A \cos(\omega(t - t_0))$$

Amplitude

Zeitpunkt des
Maximums

Eine Schwingung
pro Jahr

$$\omega = \frac{2\pi}{365,25d}$$



- Jährliche Schwingung

$$f(t) = A \cos(\omega(t - t_0))$$

$$= \underbrace{A \cos \omega t_0}_c \cos \omega t + \underbrace{A \sin \omega t_0}_s \sin \omega t$$

$$= c \cos \omega t + s \sin \omega t$$

Additionstheorem

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

- mit

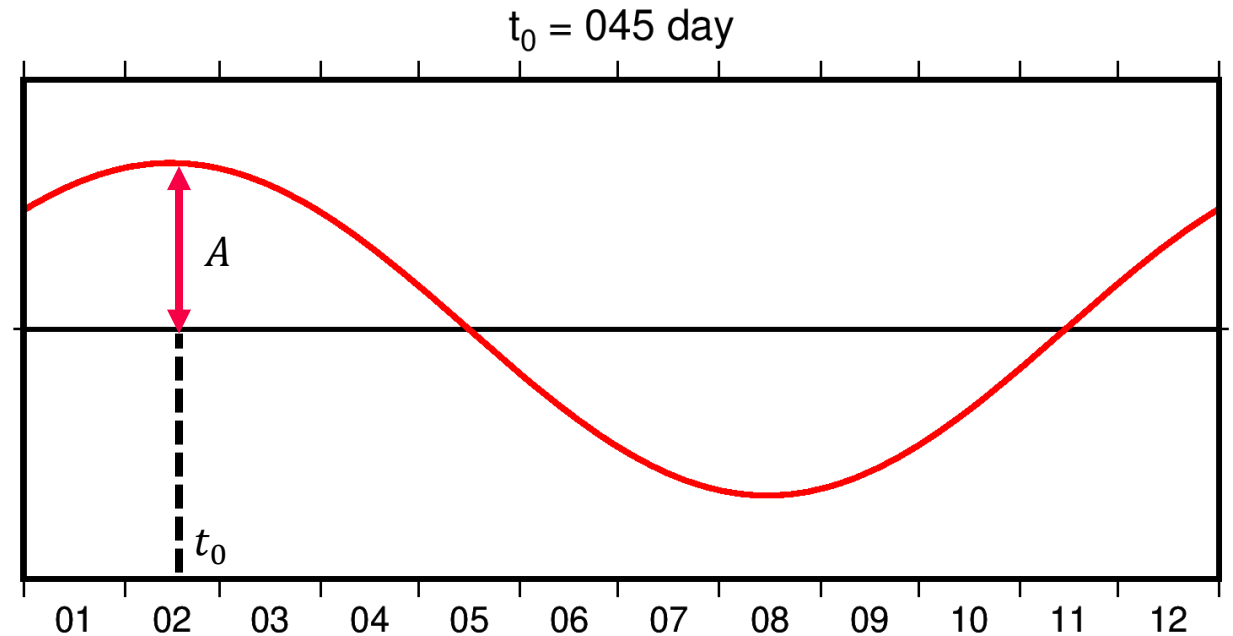
$$c = A \cos \omega t_0$$

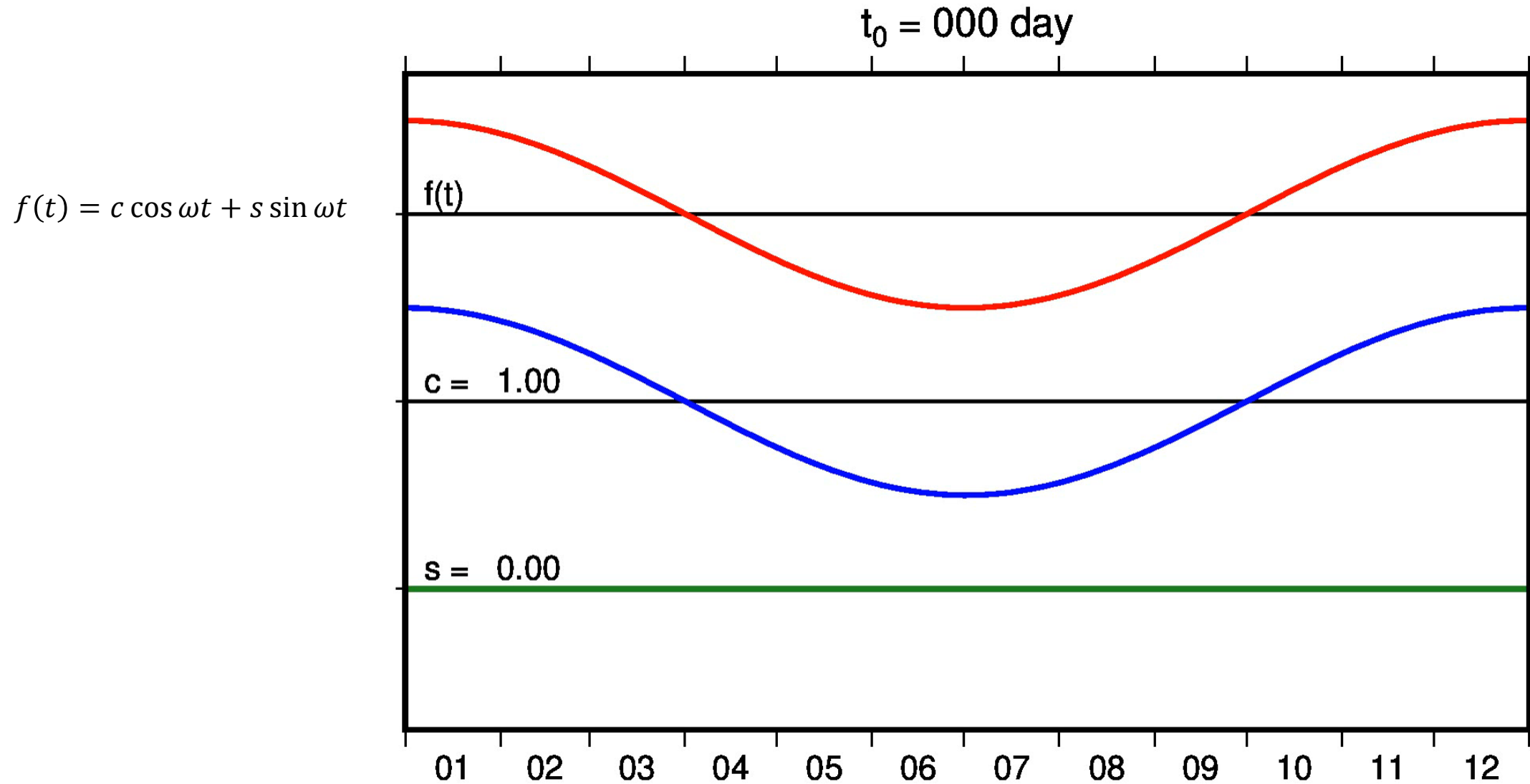
$$s = A \sin \omega t_0$$

- Amplitude und Phase

$$A = \sqrt{c^2 + s^2}$$

$$\tan \omega t_0 = \frac{s}{c}$$

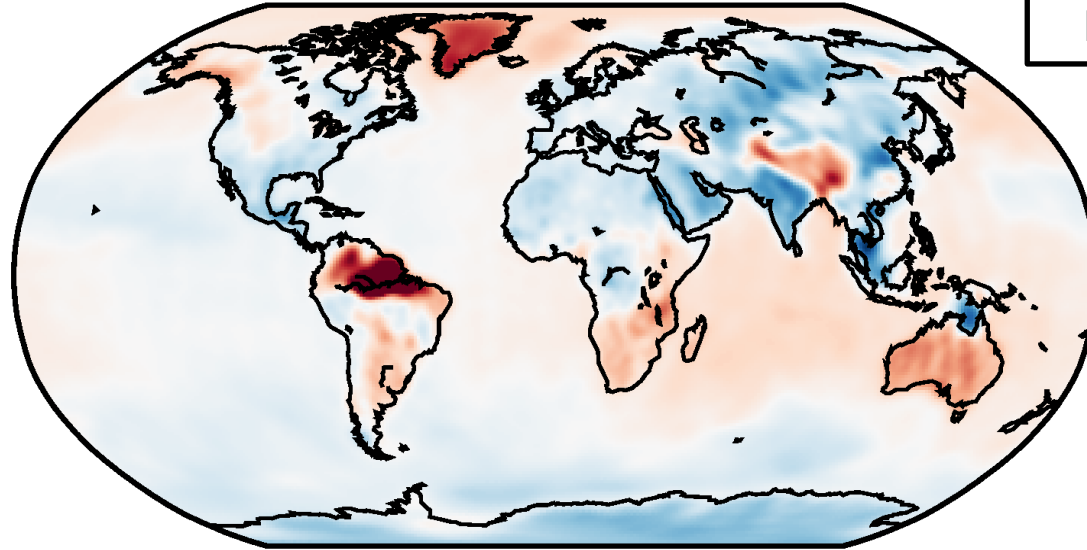




Jahresgang

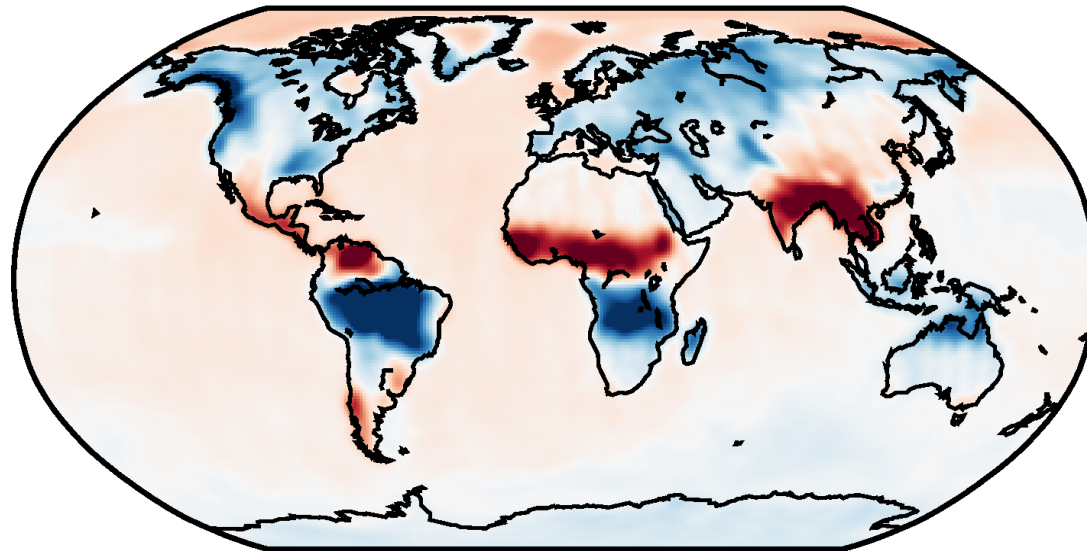
Jahresbeginn
(1. Jänner)

cos



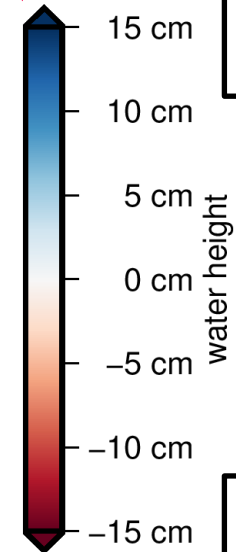
Nach ¼ Jahr
(~ 1. April)

sin



Pfeil

Es gibt auch
größere Werte



Blau: viel
Wasser

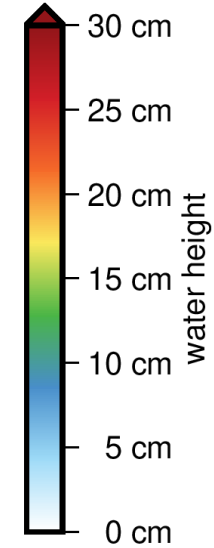
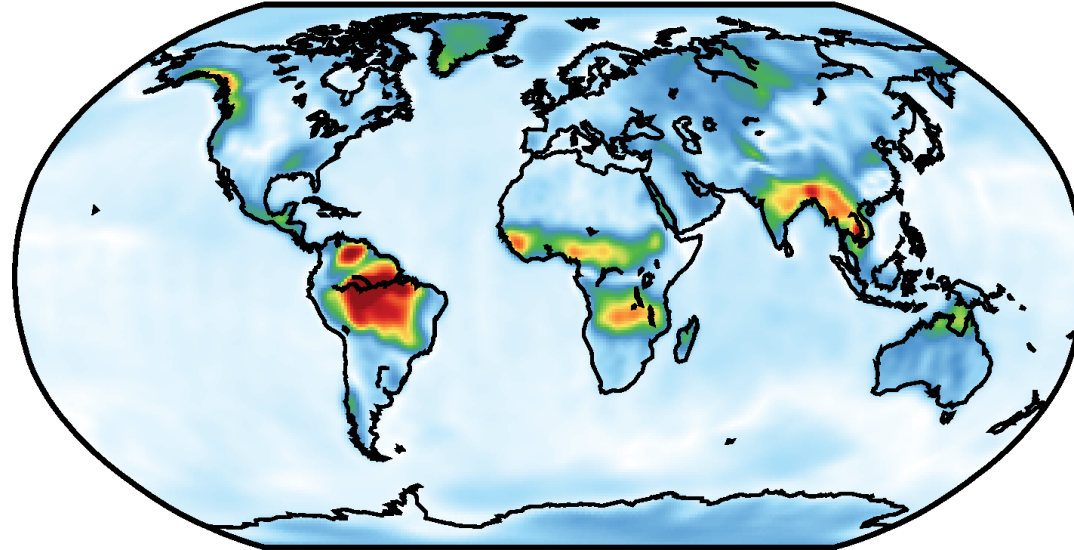
Weiß

Rot: wenig
Wasser

Beide gleich
skaliert

Amplitude

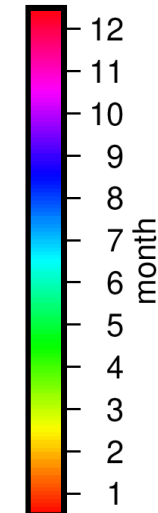
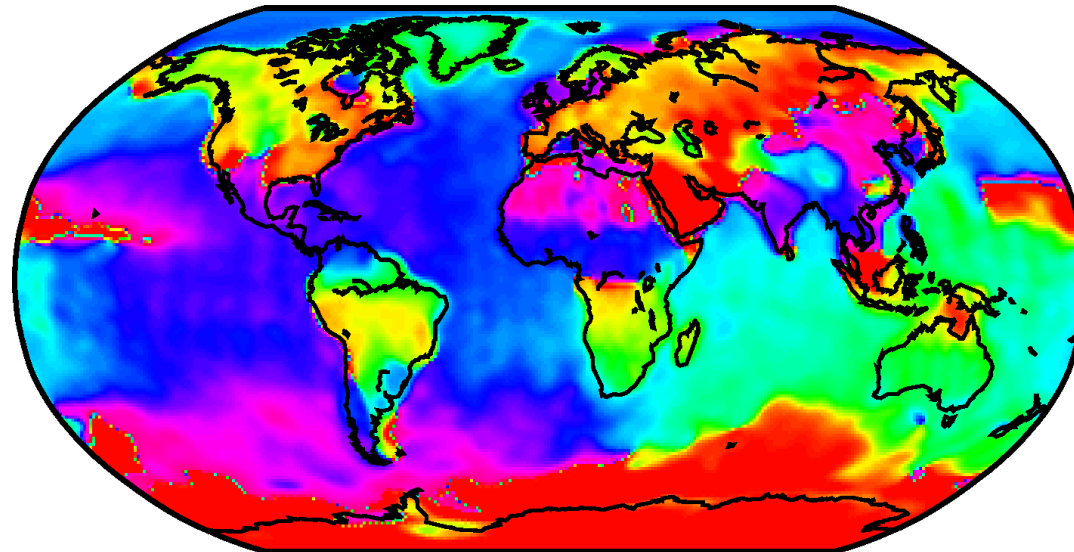
$$A = \sqrt{c^2 + s^2}$$



Phase

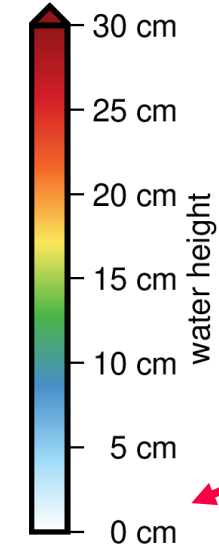
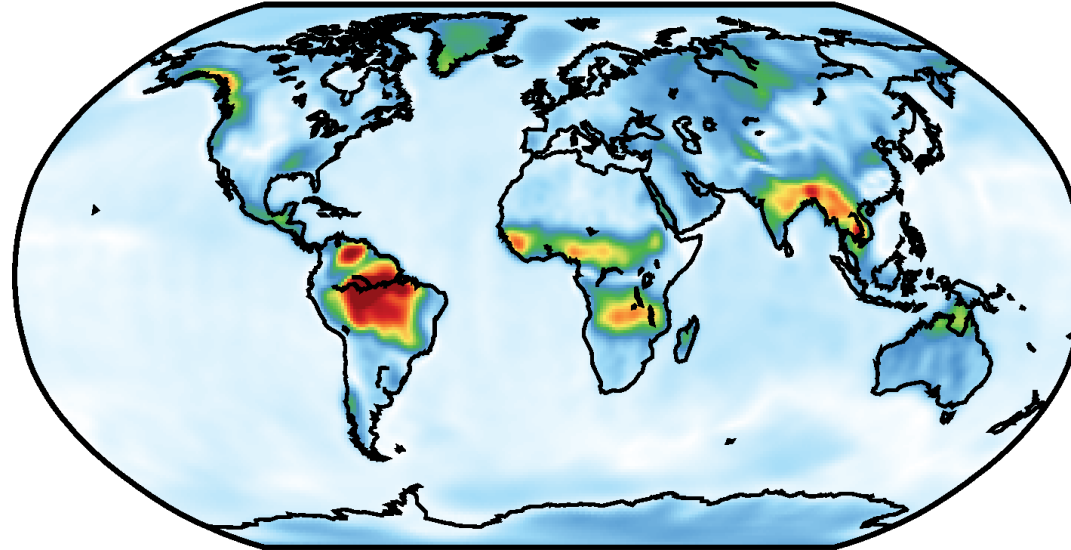
$$\varphi_0 = \frac{12}{2\pi} \text{atan2}(s, c)$$

$$\text{mod}(\varphi_0 + 12, 12)$$



Amplitude

$$A = \sqrt{c^2 + s^2}$$



Weiß: keine
Amplitude

Kein Pfeil

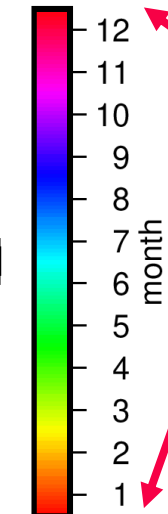
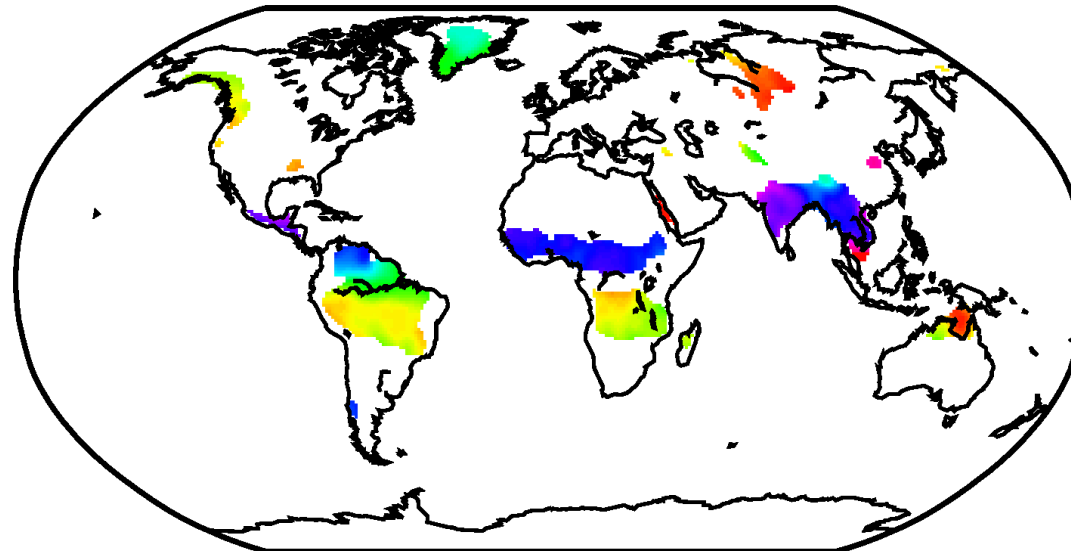
Kein Pfeil

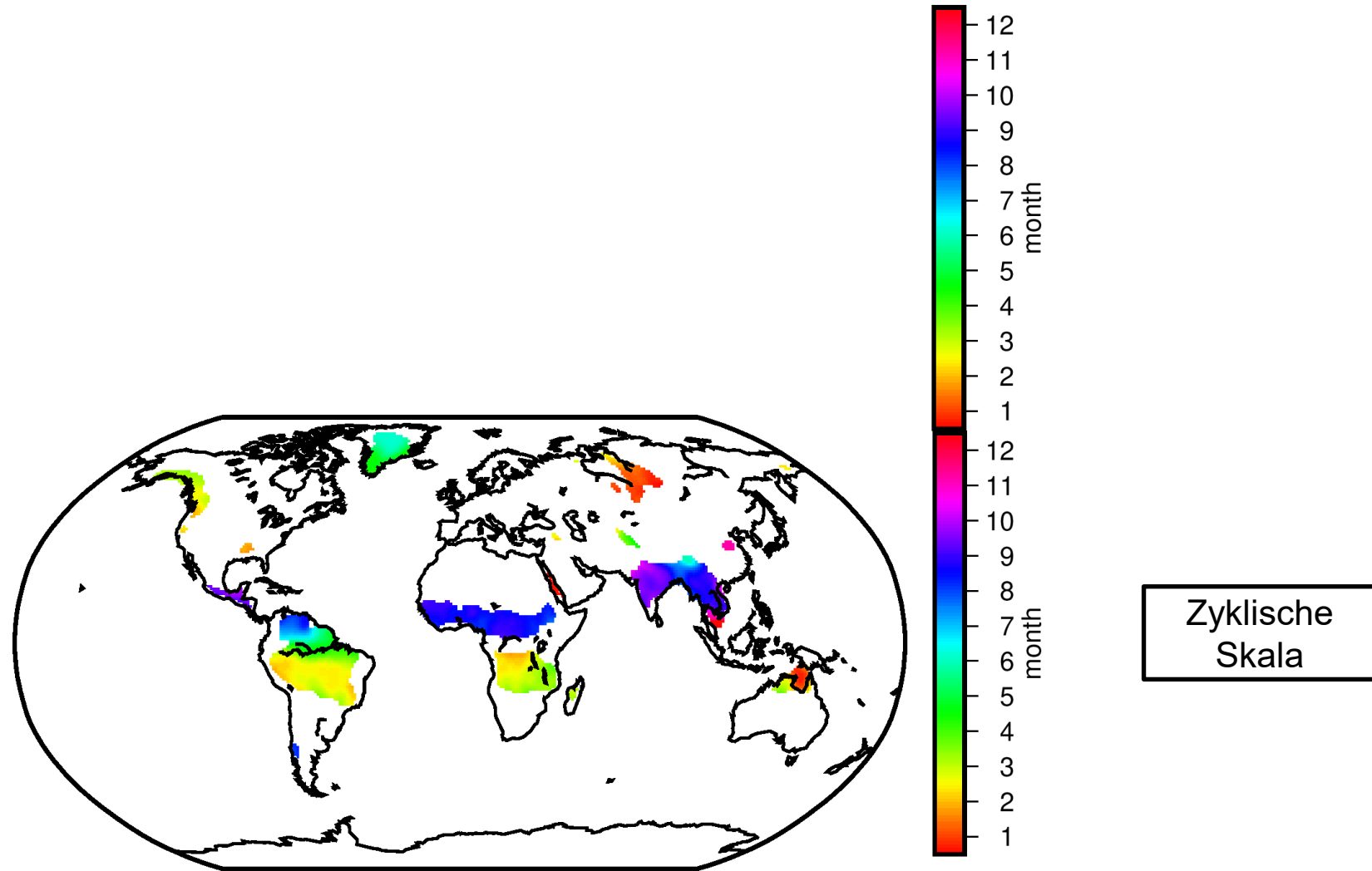
Zyklische
Skala

Phase (Ampl > 10 cm)

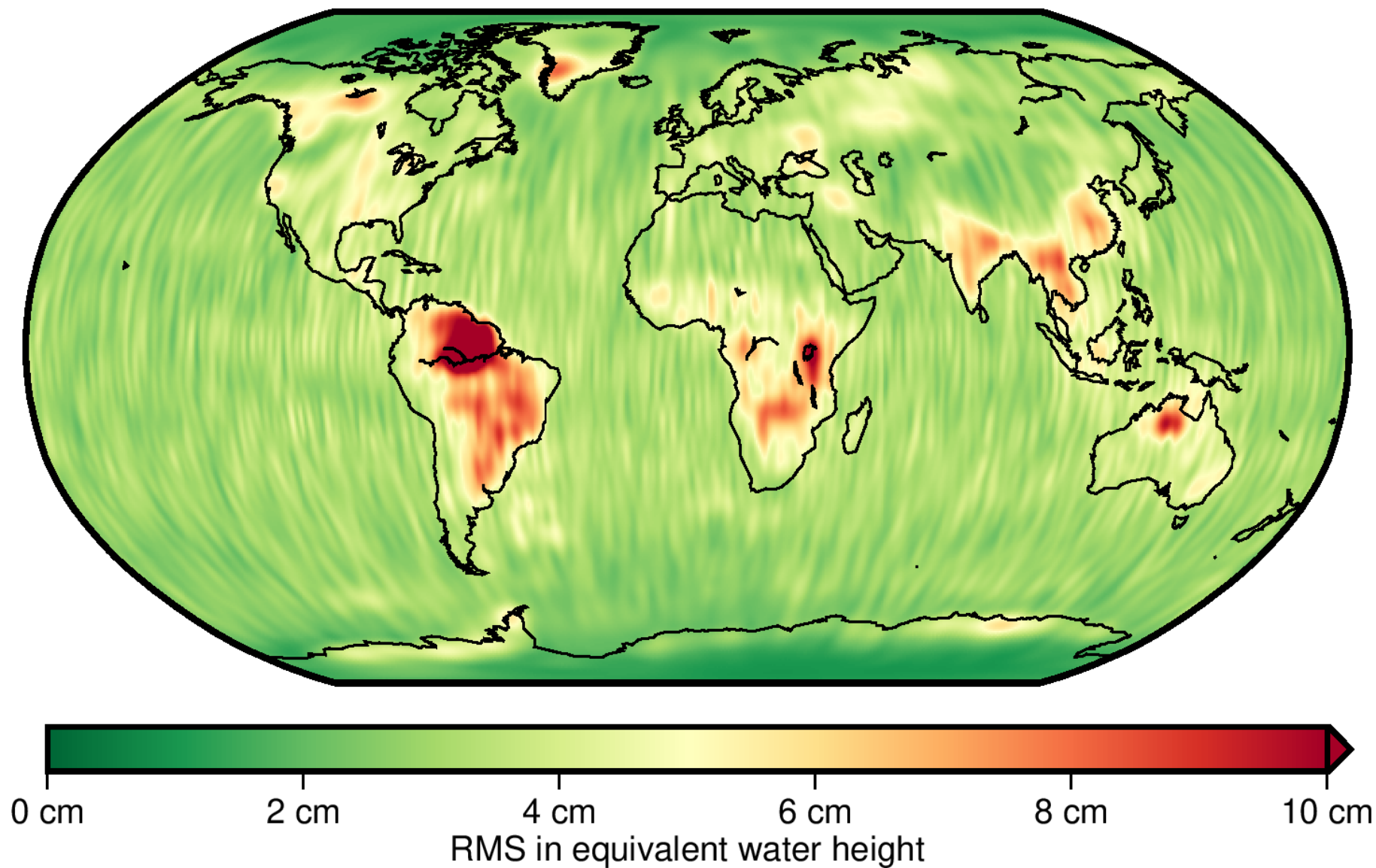
$$\varphi_0 = \frac{12}{2\pi} \text{atan2}(s, c)$$

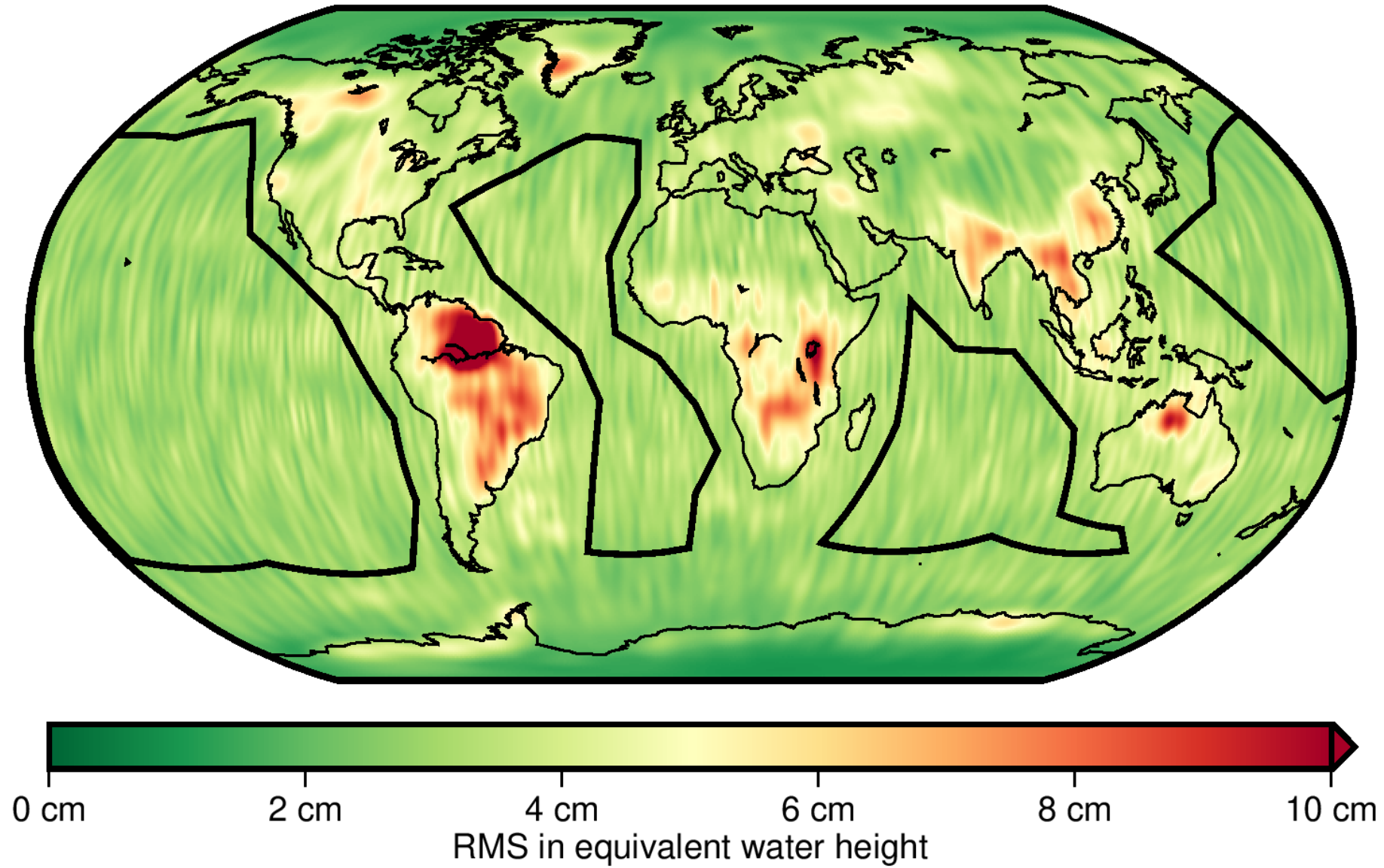
$$\text{mod}(\varphi_0 + 12, 12)$$



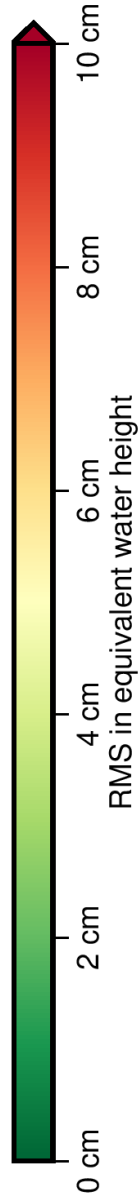
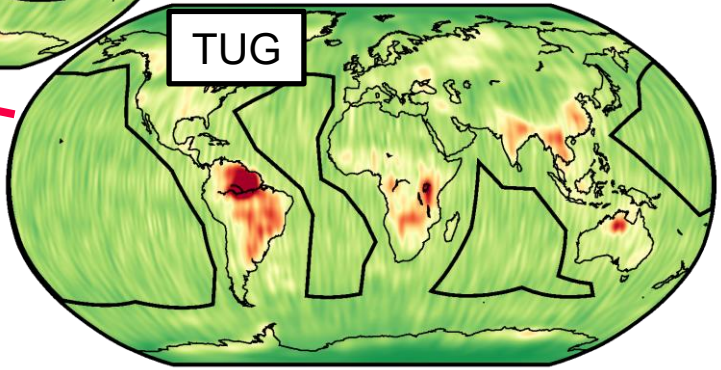
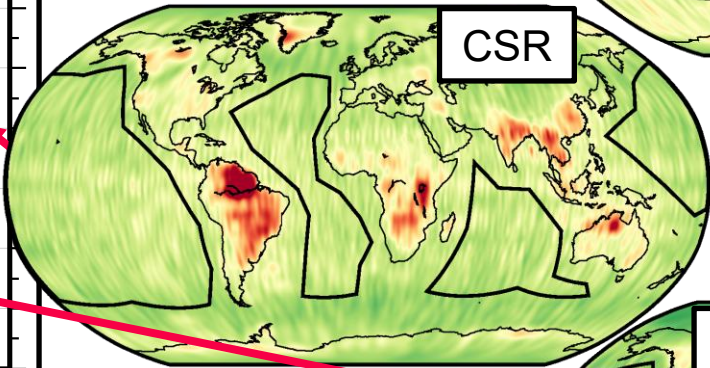
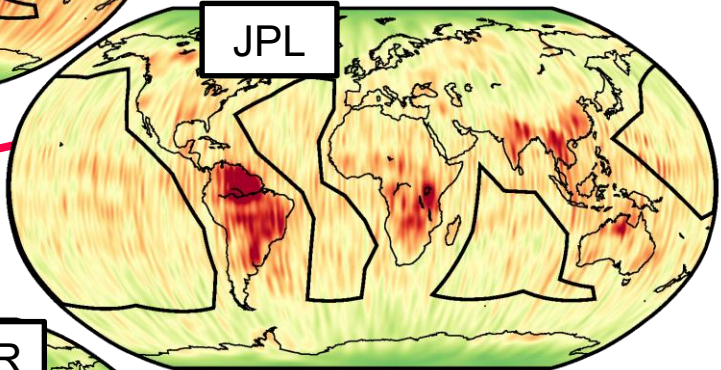
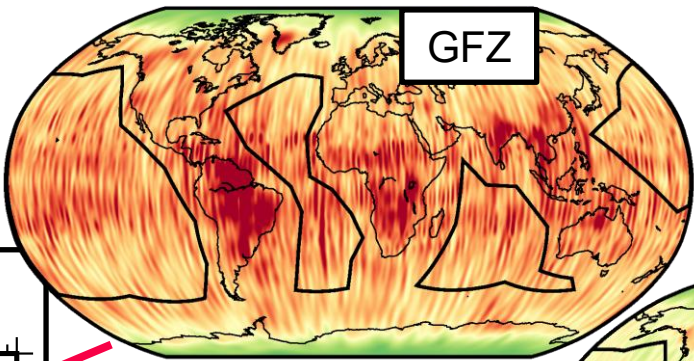
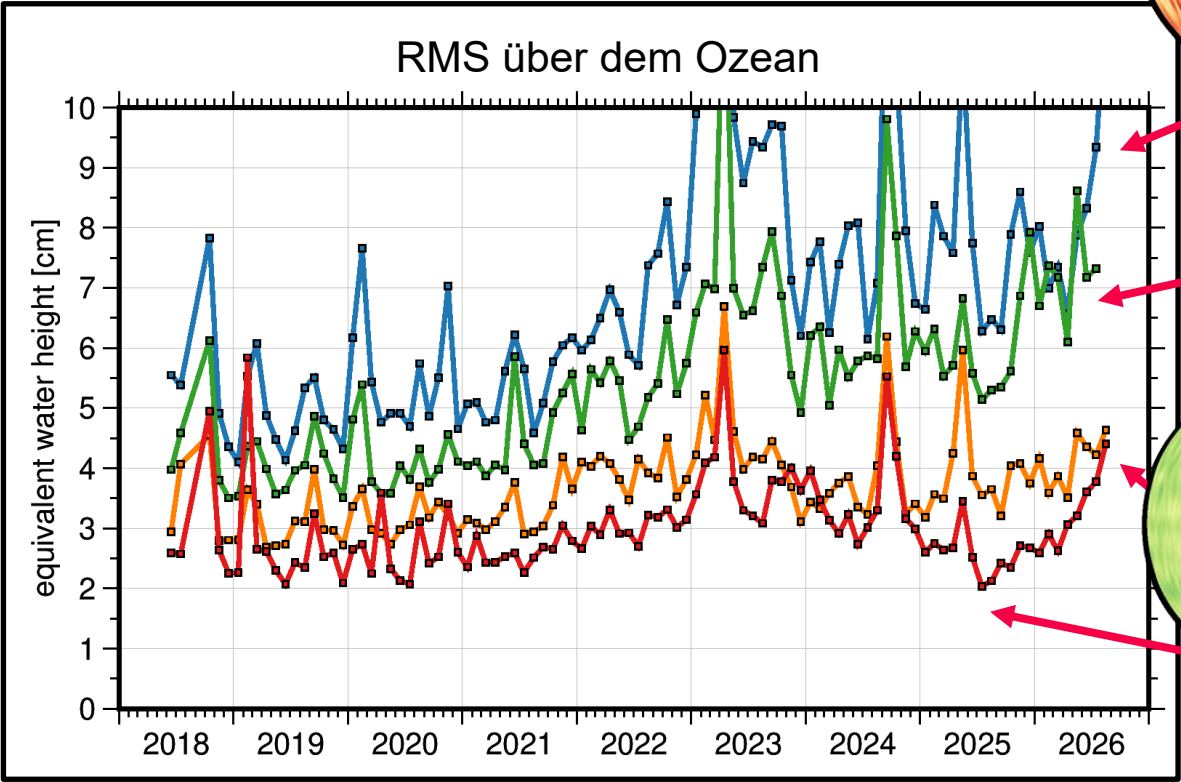


Standardabweichung pro Gitterpunkt

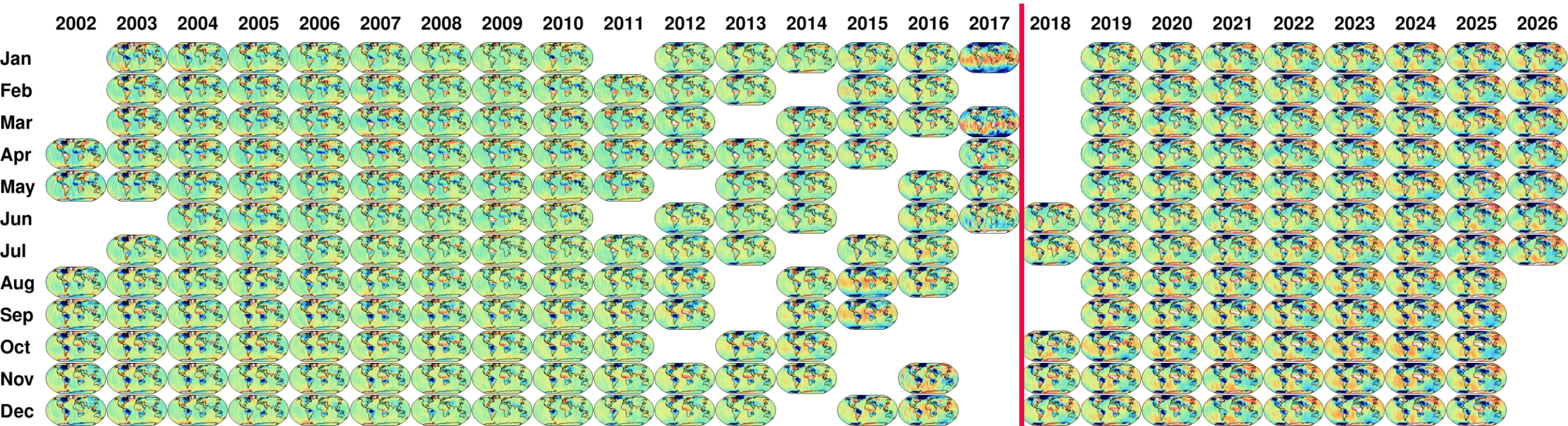




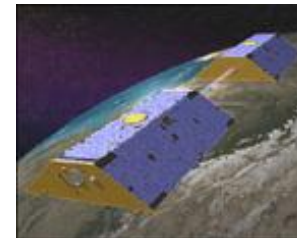
Satellitenmission GRACE-(FO)



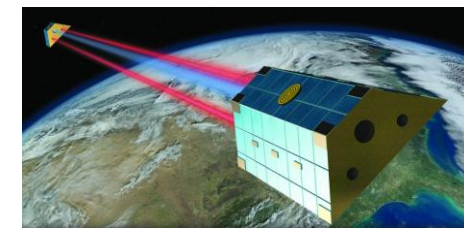
ITSG-Grace monthly gravity field solutions



GRACE



GRACE-FO



ITSG-Grace2018: Monthly and Daily solutions

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Technischer Bericht

- Nachweis der Übungsleistung
- Vorbereitung auf die Erstellung wissenschaftlicher Texte:
 - Bachelorarbeit, Masterarbeit, Dissertation, wissenschaftliche Artikel
- Sehr empfohlen: verwenden Sie LaTeX!
- Schreiben Sie den Text selber (Sie wollen etwas lernen)!

- Aussagen in wissenschaftlichen Texten sind entweder
 - Allgemein bekannt/trivial
 - Von anderen übernommen: Referenz angeben, z.B. (Mayer-Gürr et. al., 2019)
 - Eigene Interpretation: Begründung muss aus dem Text folgen

Es gibt unterschiedliche Zitierstile! In meinen Fachbereich wird wie folgt im Text zitiert:

- Ein oder zwei Autoren: (Kvas und Mayer-Gürr, 2019)
- Drei oder mehr Autoren: (Mayer-Gürr et. al., 2021)
- Zwei Artikel im gleichen Jahr: (Mayer-Gürr, 2006A und Mayer-Gürr, 2006B)

Literaturverzeichnis

- Kvas, A., Mayer-Gürr, T. (2019): GRACE gravity field recovery with background model uncertainties. *J Geod* **93**, 2543–2552. <https://doi.org/10.1007/s00190-019-01314-1>
- Mayer-Gürr, T., Behzadpour, S., Eicker, A., Ellmer, M., Koch, B., Krauss, S., Pock, C., Rieser, D., Strasser, S., Süsser-Rechberger, B., Zehentner, N., Kvas, A. (2021): GROOPS: A software toolkit for gravity field recovery and GNSS processing, *Computers & Geosciences*, Volume 155, <https://doi.org/10.1016/j.cageo.2021.104864>.
- Kvas, A., Behzadpour, S., Ellmer, M., Klinger, B., Strasser, S., Zehentner, N., & Mayer-Gürr, T. (2019): ITSG-Grace2018: Overview and evaluation of a new GRACE-only gravity field time series. *Journal of Geophysical Research: Solid Earth*, 124, 9332–9344. <https://doi.org/10.1029/2019JB017415>

- Bachelorarbeit/Masterarbeit: Generelle Gliederung
 - Titelseite
 - Abstrakt Deutsch/Englisch
 - Inhaltsverzeichnis

 - Einführung
 - Motivation/Aufgabenstellung/Fragestellung
 - Theoretische Grundlagen
 - Durchführung/Eigene Arbeiten
 - Zusammenfassung und Ausblick

 - Anhang
 - Literaturverzeichnis

- Technischer Bericht: Generelle Gliederung
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 - Zusammenfassung und Ausblick

 - Anhang
 - Literaturverzeichnis

- Ein Text besteht aus einzelnen Blöcken/Abschnitten
 - Text: Kapitel/Unterkapitel/...
 - Abbildungen
 - Tabellen
- Ein Kapitel/Unterkapitel besteht ausschließlich aus Sätzen!
 - Formeln stehen nie alleine sondern sind immer Teil eines Satzes (mit Satzzeichen)
 - Referenzen gehören zum Satz
 - Einschübe (Klammern) gehören zum Satz
 - Abbildungen und Tabellen sind unabhängige Blöcke und nicht Teil des Kapitels (dazu später mehr)
- Referenzen:
 - Dies ist ein Satz, siehe Mayer-Gürr et. al., 2019.
 - Dies ist ein Satz (Mayer-Gürr et. al., 2019). FALSCH: Dies ist ein Satz. (Mayer-Gürr et. al., 2019)
- Einschübe:
 - Dies ist ein Satz (zusätzliche Bemerkung). FALSCH: Dies ist ein Satz. (zusätzliche Bemerkung)
- Formeln:
 - nächste Folie

Wissenschaftlicher Text

- Auszug aus meiner Dissertation, Mayer-Gürr 2006:

Gleichungen referenzieren

- „Wenn (2.31) in (2.35) einsetzt folgt: ...“
- Da die Formel zum Satz gehört, muss man im gleichen Satz nicht auf diese Formel referenzieren

Damit erhält man die Schätzung der Parameter

$$\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{l}. \quad (2.9)$$

Formeln gehören zum Satz (mit Satzzeichen)

Das Dach auf der Lösung soll darauf hindeuten, dass es sich nicht um die wahren Parameter $\mathbf{x}$ handelt, sondern nur um eine bestmögliche Schätzung aus den gemessenen Beobachtungen. Aus den geschätzten Parametern lassen sich auch die Residuen schätzen

$$\hat{\mathbf{e}} = \mathbf{l} - \mathbf{A}\hat{\mathbf{x}}, \quad (2.10)$$

Numerierung der Formeln

und die geschätzten (fehlerfreien) Beobachtungen berechnen

$$\hat{\mathbf{l}} = \mathbf{l} - \hat{\mathbf{e}}. \quad (2.11)$$

Aus den geschätzten Residuen lässt sich außerdem die Genauigkeit der Messungen abschätzen. Die Schätzung der Varianz der Beobachtungen ergibt sich zu

$$\hat{\sigma}^2 = \frac{\hat{\mathbf{e}}^T \hat{\mathbf{e}}}{n - m}, \quad (2.12)$$

Variablen *kursiv*

mit der Anzahl der Beobachtungen n und der Anzahl der Parameter m . Eine Herleitung dieser Formel erfolgt später.

- Alle Symbole sollten in den Formeln und im Text gleich dargestellt werden (z.B. Variablen kursiv), (Latex: $\$n\$$)
- Alle Symbole müssen erklärt werden
- Beim Programmieren ist die Multiplikation ein $*$ in der Mathematik ein $\cdot$ ($\backslash cdot$), wird aber meist weggelassen

- Falsch: $\hat{\mathbf{x}} = (\mathbf{A}^T * \mathbf{A})^{-1} * \mathbf{A}^T * \mathbf{l}$
- Richtig (aber nicht gut): $\hat{\mathbf{x}} = (\mathbf{A}^T \cdot \mathbf{A})^{-1} \cdot \mathbf{A}^T \cdot \mathbf{l}$
- Schön: $\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{l}$

- Matrizen und Vektoren können (müssen nicht) fett dargestellt werden (aber einheitlich)

- Zahlen: Unterschied Programmieren und Mathematik

- Falsch: $1.048\text{e-}05$
- Richtig: $1,048 \cdot 10^{-5} \text{ m}^2$ 

Einheiten nicht vergessen

- Passende Anzahl von Ziffern

$$x = 5,0032 \text{ m} \pm 3.0 \text{ mm}$$

Eine Ziffer mehr als Genauigkeit

Bei Genauigkeiten reichen 2 Ziffern

- Abbildungen/Tabellen sind nicht Teil des Texts, sondern bilden extra Blöcke mit eigener Unterschrift und Nummer
- Häufig am Beginn oder Ende einer Seite
- Abbildungen mit Unterschrift stehen für sich allein und sollten selbsterklärend sein

- Abbildungen müssen im Text referenziert werden
- Interpretation gehört in den Text
- Kapitel/Abschnitte nur mit Abbildungen sind sinnlos (Leeres Kapitel, da Abb. nicht Teil des Kapitels sind)

Abbildungen

- Abbildungen/Tabellen sind nicht Teil des Texts, sondern bilden extra Blöcke mit eigener Unterschrift und Nummer
- Häufig am Beginn oder Ende einer Seite
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- Kapitel/Abschnitte nur mit Abbildungen sind sinnlos (Leeres Kapitel, da A)

Table 3 Block-specific statistics of GPS L1/L2 phase pattern differences between ground calibrated and observed patterns

Block	LMB-TUG	IGS-TUG
IIR-A	1.9	2.2
IIR-B/IIR-M	1.1	1.6
IIF	1.1	1.8
III	1.3	1.3

All values are given in mm and represent median values of the respective satellite-specific RMS differences on a triangular vertex grid. LMB = Lockheed Martin/Boeing

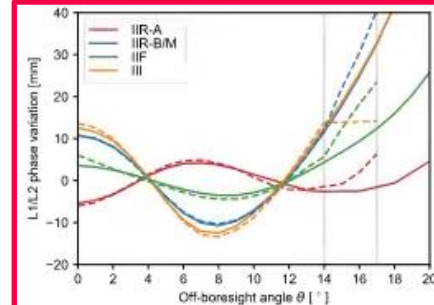


Fig. 5 Block- and azimuth-averaged L1/L2 phase patterns from manufacturer calibrations (solid lines) and corresponding values of the IGS20.atx antenna model (dashed lines). For proper comparison, a PCO and bias contribution has been removed from each data set based on equal weighting on a 1° grid up to 14° off-boresight angle. Gray lines mark the limits of the IGS model covered by Earth-based observations and the extension derived from LEO satellite measurements

2D transmit antenna phase patterns in precise GPS processing.

Despite the subtle deviations between ground calibrations and in-flight antenna characteristics, the manufacturer data offer unique advantages over the observed patterns. On the one hand, independent patterns are provided for each frequency band, which supports a consistent observation modeling in an uncombined processing scheme and facilitates a fully consistent use of triple-frequency (L1, L2, L5) observations. On the other hand, the ground calibrations cover an off-boresight angle of at least 23° and can seamlessly be applied for spaceborne platforms at altitudes of up to almost 10,000 km.

For illustration, Fig. 5 shows the variation of block- and azimuth-averaged phase patterns with off-boresight angle for the ionosphere-free L1/L2 combination. In accordance with the previous discussion, a good consistency of manufacturer calibrations and the IGS antenna model can be recognized over the range of off-boresight angles covered by observations with the IGS station network. Except for a 2.5 mm

difference in the IIF antenna pattern at $\theta = 0^\circ$, the agreement is always better than 1.1 mm. As may be recognized from the nonzero slope of the IIF phase variations in boresight direction, the discrepancy can readily be attributed to a deficiency of the IGS pattern estimates, which was first introduced into the IGS20.atx antenna model and retained since then. Based on GPS tracking from LEO satellites with altitudes of up to 1340 km, extensions of the IIR-A, IIR-B/M, and IIF antenna patterns up to $\theta = 17^\circ$ were derived in Jäggi et al. (2010) and Jäggi et al. (2012), which were subsequently adopted for the IGS20.atx antenna model. For GPS III satellites, in contrast, no spaceborne observations were available when establishing the first IGS phase pattern model for these satellites, and a constant extension was adopted instead.

Improved extensions consistent with the other blocks were later presented in Conrad et al. (2023) and Dilssner et al. (2023), but have not been incorporated into the IGS antenna model so far. However, a very poor match of the IGS phase pattern extensions is also evident for IIR-A, IIR-B/M, and IIF satellites, which increases to roughly 10 mm at $\theta = 17^\circ$. This mismatch can primarily be attributed to the fact that the IGS values beyond $\theta = 14^\circ$ do not represent absolute phase pattern, but have been determined relative to two Block IIA satellites. For these, a constant phase variation was arbitrarily assumed to allow for the separation of the unknown GPS transmit antenna patterns and LEO receive antenna phase patterns in that region (Jäggi et al. 2010, 2012). Among others, this results in a sudden change of slope of the IGS phase patterns at 14° , whereas the manufacturer calibrations exhibit a smooth variation with a continuous derivative at this point.

Due to the fact that the patterns in Fig. 5 are minimized over the Earth coverage zone ($\theta \leq 14^\circ$), a notable increase outside this region may be observed. This is most pronounced for the IIR-B/M and GPS III satellites, where values of 65 mm are attained at off-boresight angles of 20° for the azimuth-averaged L1/L2 patterns. In addition, notable azimuth variations may arise outside the Earth coverage zone of the transmit antennas. They are most pronounced for the IIR-A and IIF satellites and can attain amplitudes of about 10 mm level near $\theta = 20^\circ$. Evidently, use of azimuth-mean patterns is discouraged in this region and the use of 2D patterns is clearly advisable for the precise modeling of spaceborne GNSS observations beyond the common altitude range of current LEO missions.

4.2 Phase center offsets

For proper GNSS observation modeling, a consistent set of transmit antenna data is required, which comprises, on the one hand, the offset of the phase center from the center-of-mass and, on the other hand, the phase variations with respect to the phase center. Evidently, identical definitions of the phase center need to be applied for both parts of the

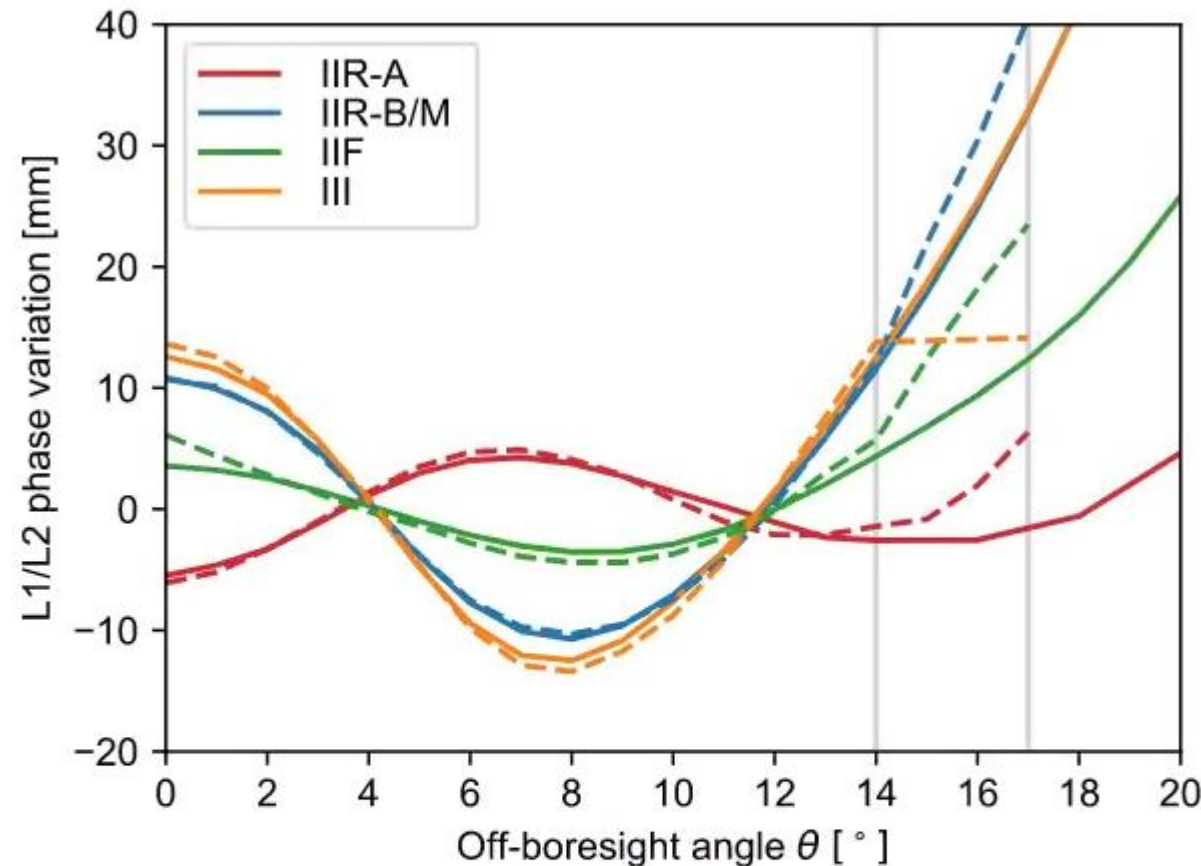


Fig. 5 Block- and azimuth-averaged L1/L2 phase patterns from manufacturer calibrations (solid lines) and corresponding values of the igs20.atx antenna model (dashed lines). For proper comparison, a PCO and bias contribution has been removed from each data set based on equal weighting on a 1° grid up to 14° off-boresight angle. Gray lines mark the limits of the IGS model covered by Earth-based observations and the extension derived from LEO satellite measurements

Table 3 Block-specific statistics of GPS L1/L2 phase pattern differences between ground calibrated and observed patterns

Block	LMB-TUG	IGS-TUG
IIR-A	1.9	2.2
IIR-B/IIR-M	1.1	1.6
IIF	1.1	1.8
III	1.3	1.3

All values are given in mm and represent median values of the respective satellite-specific RMS differences on a triangular vertex grid. LMB = Lockheed Martin/Boeing

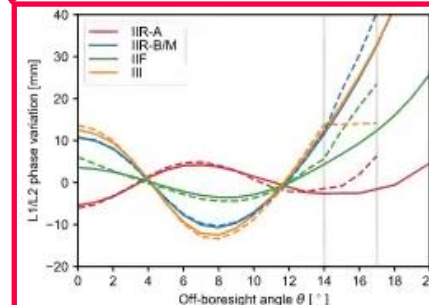


Fig. 5 Block- and azimuth-averaged L1/L2 phase patterns from manufacturer calibrations (solid lines) and corresponding values of the igs20.atx antenna model (dashed lines). For proper comparison, a PCO and bias contribution has been removed from each data set based on equal weighting on a 1° grid up to 14° off-boresight angle. Gray lines mark the limits of the IGS model covered by Earth-based observations and the extension derived from LEO satellite measurements

2D transmit antenna phase patterns in precise GPS processing.

Despite the subtle deviations between ground calibrations and in-flight antenna characteristics, the manufacturer data offer unique advantages over the observed patterns. On the one hand, independent patterns are provided for each frequency band, which supports a consistent observation modeling in an uncombined processing scheme and facilitates a fully consistent use of triple-frequency (L1, L2, L5) observations. On the other hand, the ground calibrations can seamlessly be extended to larger off-boresight angles of up to 20° for the azimuth-averaged L1/L2 patterns. In addition, notable azimuth variations may arise outside the Earth coverage zone of the transmit antennas. They are most pronounced for the IIR-A and IIF satellites and can attain amplitudes of about 10 mm level near $\theta = 20^\circ$. Evidently, use of azimuth-mean patterns is discouraged in this region and the use of 2D patterns is clearly advisable for the precise modeling of spaceborne GNSS observations beyond the common altitude range of current LEO missions.

difference in the IIF antenna pattern at $\theta = 0^\circ$, the agreement is always better than 1.1 mm. As may be recognized from the nonzero slope of the IIF phase variations in boresight direction, the discrepancy can readily be attributed to a deficiency of the IGS pattern estimates, which was first introduced into the igs08.atx antenna model and retained since then. Based on GPS tracking from LEO satellites with altitudes of up to 1340 km, extensions of the IIR-A, IIR-B/M, and IIF antenna patterns up to $\theta = 17^\circ$ were derived in Jäggi et al. (2010) and Jäggi et al. (2012), which were subsequently adopted for the igs20.atx antenna model. For GPS III satellites, in contrast, no spaceborne observations were available when establishing the first IGS phase pattern model for these satellites, and a constant extension was adopted instead.

Improved extensions consistent with the other blocks were later presented in Conrad et al. (2023) and Dilssner et al. (2023), but have not been incorporated into the IGS antenna model so far. However, a very poor match of the IGS phase pattern extensions is also evident for IIR-A, IIR-B/M, and IIF satellites, which increases to roughly 10 mm at $\theta = 17^\circ$. This mismatch can primarily be attributed to the fact that the IGS values beyond $\theta = 14^\circ$ do not represent absolute phase pattern, but have been determined relative to two Block IIA satellites. For these, a constant phase variation was arbitrarily assumed to allow for the separation of the unknown GPS transmit antenna patterns and LEO receive antenna phase patterns in that region (Jäggi et al. 2010, 2012). Among others, this results in a sudden change of slope of the IGS phase patterns at 14° , whereas the manufacturer calibrations exhibit a smooth variation with a continuous derivative at this point.

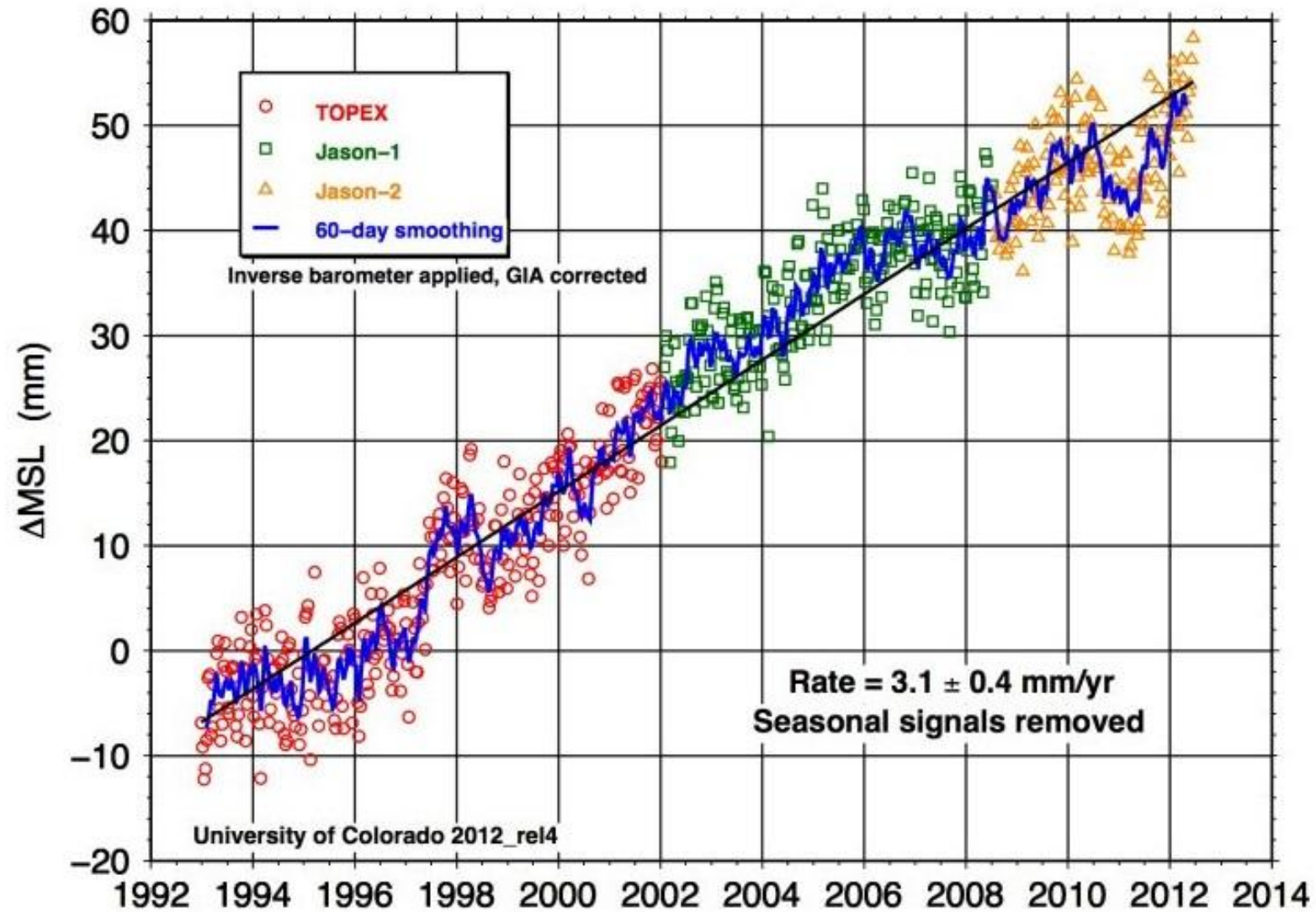
Due to the fact that the patterns in Fig. 5 are minimized over the Earth coverage zone ($\theta \leq 14^\circ$), a notable increase outside this region may be observed. This is most pronounced for the IIR-B/M and GPS III satellites, where values of 65 mm are attained at off-boresight angles of 20° for the azimuth-averaged L1/L2 patterns. In addition, notable azimuth variations may arise outside the Earth coverage zone of the transmit antennas. They are most pronounced for the IIR-A and IIF satellites and can attain amplitudes of about 10 mm level near $\theta = 20^\circ$. Evidently, use of azimuth-mean patterns is discouraged in this region and the use of 2D patterns is clearly advisable for the precise modeling of spaceborne GNSS observations beyond the common altitude range of current LEO missions.

4.2 Phase center offsets

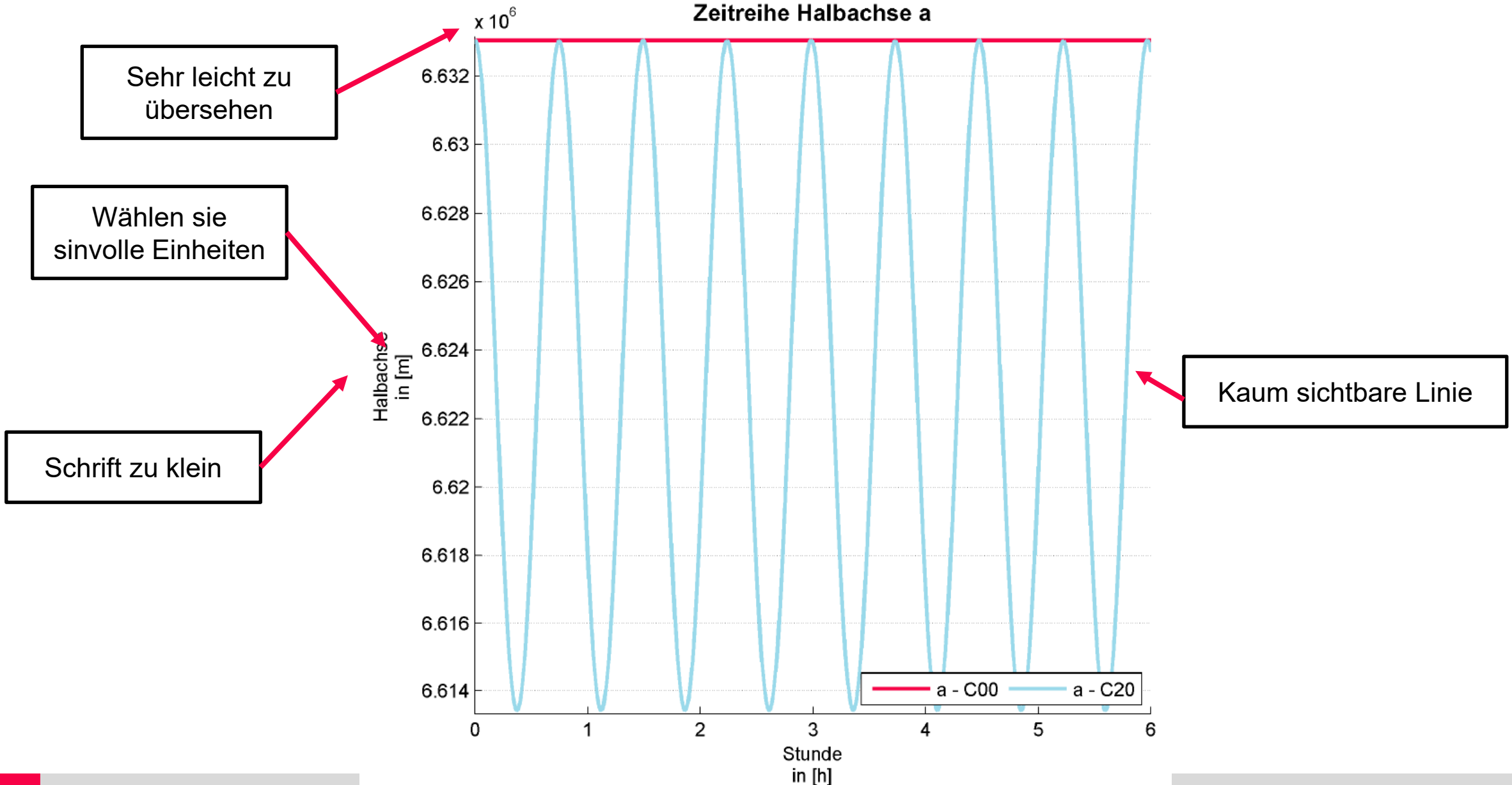
For proper GNSS observation modeling, a consistent set of transmit antenna data is required, which comprises, on the one hand, the offset of the phase center from the center-of-mass and, on the other hand, the phase variations with respect to the phase center. Evidently, identical definitions of the phase center need to be applied for both parts of the

Wenn nötig, können das mehrere Zeilen sein

- Abbildungen/Tabellen sind nicht Teil des Texts, sondern bilden extra Blöcke mit eigener Unterschrift und Nummer
- Häufig am Beginn oder Ende einer Seite
- Abbildungen mit Unterschrift stehen für sich allein und sollten selbsterklärend sein
- Abbildungen müssen im Text referenziert werden
- Interpretation gehört in den Text
- Kapitel/Abschnitte nur mit Abbildungen sind sinnlos (Leeres Kapitel, da Abb. nicht Teil des Kapitels sind)
- Legenden, Achsenbeschriftungen sollten eine ähnliche Schriftgröße wie die Bildunterschrift haben



Abbildungen (schlechtes Beispiel)



■ Technischer Bericht: Generelle Gliederung

■ Titelseite

- Einführung: Aufgabenstellung
- Theoretische Grundlagen
- Durchführung/Eigene Arbeiten
- Ergebnisse
 - Mit Interpretation
- Zusammenfassung und Ausblick
- Anhang
- Literaturverzeichnis

Für diesen Bericht.

Benötigte Formeln aus der Vorlesung,
Verweis auch die Vorlesung

Anwendung der Formeln:

- Wie ist der Vektor l und x aufgebaut?
- Was steht in der Designmatrix?
- Dimensionen?

Was habe ich getan? Was könnte man noch tun?